# A Multiscale Streamline Method for Inversion of Production Data

Vegard R. Stenerud  $^{*,1}$ 

Department of Mathematical Sciences, NTNU, N-7491 Trondheim, Norway

# Knut–Andreas Lie<sup>2</sup>

Department of Applied Mathematics, SINTEF, P.O. Box 124 Blindern, N-0314 Oslo, Norway

#### Abstract

We propose two improvements to a recent streamline method by Wang and Kovscek for inversion of production data. The key idea of the Wang–Kovseck method is to associate increments in fractional flow-curves (water-cut) with breakthrough of individual streamlines and match breakthrough times of each streamline by adjusting the effective streamline permeabilities. The perturbations in effective streamline permeabilities are given by a linear system, which can be solved in a decoupled fashion under additional simplifying assumptions. Finally, the permeabilities perturbations defined along streamlines are mapped onto the underlying simulation grid, typically using a geostatistical algorithm to constrain the corresponding corrections to the geological model to prior geological data.

Our first improvement is to model the flow in each streamline independently using real time, instead of using Dykstra–Parsons' algorithm for all streamlines connected to a producer-injector pair. This way, there is no coupling between individual streamlines, and permeability modifications can be obtained directly. Our approach uses less approximations, enables extension of the formulation to include gravity, and enables history matching of porosity. Three synthetic test cases show that this approach gives a better match and a faster convergence.

Our second point is to use a multiscale inversion process, where the reservoir parameters are matched on a hierarchy of recursively coarsened grids. Two synthetic test cases demonstrate that this approach captures the large-scale trends of the reservoir parameters more accurately. The proposed approach has proven robust in the sense that it is able to capture structures of the permeability field on the basis of limited information.

Key words: History matching, Streamlines, Permeability, Multiscale

# 1 Introduction

Obtaining a reliable history match is an ill-posed and time-consuming exercise for reservoir engineers. In this paper we consider one component of history matching, namely how to modify permeabilities (and/or porosities) to match observed production data. The process for history matching permeabilities typically consists of two basic steps: (1) modification of grid-block permeabilities, and (2) forward simulation of fluid responses to validate the accuracy/correctness of a given permeability distribution. History matching real-life reservoirs typically requires numerous flow simulations, which often makes forward simulation the most time-consuming part of a history match.

Streamline simulation (Datta-Gupta and King, 1995; Thiele, 2005) is a complementary technology for flow simulation in petroleum reservoirs. Compared to traditional finite-difference simulators, streamline simulation offers unparallelled computational efficiency for simulating reservoir responses for large and complex geomodels and for flow scenarios dominated by wells, fluid mobilities, and heterogeneity in the rock properties. Replacing a conventional finite-difference solver by a much faster streamline solver may therefore drastically reduce the computational time, thereby allowing more frequent model updates and possibly also models with a larger number of gridblocks.

However, the most promising use of streamlines in history matching has come from their ability to locate regions in the reservoir that may contain potential sources for mismatch in production data. Several authors have exploited streamlines to develop efficient inversion methods using a sensitivity approach, in which one needs to compute gradients of production characteristics with respect to the geological parameters; see e.g., Gautier et al. (2001, 2004); Vasco and Datta-Gupta (1999); Vasco et al. (1999); Wen et al. (2003).

As an alternative, novel inversion methods can be developed using two types of data that are not offered in conventional simulators: flow paths (the streamlines) and time-of-flight. The streamlines give a natural way to delineating the reservoir volume to be matched. Emanuel and Milliken (1998) and Milliken et al. (2000) use streamlines to define subregions, in which subsequent changes in grid properties can be performed manually (or semi-automatically) by the

<sup>\*</sup> Corresponding author. Fax: +47 73593524

Email addresses: vegarste@math.ntnu.no (Vegard R. Stenerud),

Knut-Andreas.Lie@sintef.no (Knut-Andreas Lie).

URLs: http://www.math.ntnu.no/ vegarste (Vegard R. Stenerud),

http://folk.uio.no/kalie (Knut-Andreas Lie).

<sup>&</sup>lt;sup>1</sup> Funded by the Uncertainty in Reservoir Evaluation (URE) program at the Norwegian University of Science and Technology.

 $<sup>^{2}</sup>$  Funded by the Research Council of Norway through grant no. 152732/S30.

reservoir engineer to match production data.

Wang and Kovscek (2000) use streamlines as a natural parameterisation of the reservoir and modify effective properties along streamlines to increase/decrease breakthrough times (computed from the time-of-flights), thereby reducing the mismatch between observed and calculated fractional flows, pressure drops, and total flow rates. The modified effective properties would then be mapped back to individual grid cells in the underlying geological grid model and a flow simulation performed to check the match for the new permeability estimate. This procedure is repeated until the history match is converged. Although this approach is quite robust in the sense that a reasonable history match can be obtained from a small data set, modifying grid properties directly along flow paths may introduce artifacts and violate geological constraints. To improve the predictive powers and impose geological consistency in every step of the inversion, Caers and coworkers apply the modified effective streamline properties to constrain geostatistical algorithms; see Caers (2003); Caers et al. (2002, 2004); Gross et al. (2004). Moreover, as an alternative to perturbing effective properties associated with a single streamline, streamlines and time-of-flight can be used to perform corrections on all streamlines associated with a single well, an injector-producer pair, or on all wells in a reservoir.

In this paper, we go back to the original method of Wang and Kovscek (2000) and present two possible improvements. Our first point is that one can simplify the calculation of modified streamline permeabilities. Rather than using Dykstra–Parsons' algorithm to model all streamlines connected to a well as the relative motion of saturation fronts in a set of non-communicating layers, resulting in a linear system for the perturbations in effective permeabilities, we model each streamline independently using real time and obtain directly a set of (simple) algebraic relations between the effective parameters of the streamline and the mismatch in production data. This approach generally gives a better match and faster convergence and uses less approximations and assumptions on the mobility ratios than Wang and Kovscek in their inversion method. Moreover, our new formulation is more flexible and can easily be extended to include gravity (and possibly also more complex flow physics). Finally, since the new formulation only changes the way the modified streamline permeabilities are calculated, the method can immediately be applied within the geostatistical framework developed by Caers (2003); Caers et al. (2002, 2004; Gross et al. (2004).

Our second point is that the use of a multiscale inversion process may speed up the convergence and improve the quality of the history match. We use a family of hierarchically refined grids that are formed by coarsening an underlying fine geogrid for the desired permeability. Starting with the coarsest grid, we match production data using effective permeabilities and map the perturbed streamline permeabilities back onto the coarse grid. Next, an interpolation of the coarse-grid permeabilities is used as initial value for a new match on the next hierarchically refined grid, and so on until the finest grid is reached or a sufficiently good match is obtained. There are several advantages to this approach. First, the refinement level of the inverted permeability field will correspond to the resolution of the production data, and one reduces spurious effects due to over-parameterisation. Second, decomposing the inverse problem by scale will generally improve the identification of large-scale heterogeneity structures. Finally, since only a few parameters are matched on the coarser grids, the inversion process will be much faster than using a direct fine-scale streamline inversion. A similar approach has previously been introduced by Yoon et al. (1999) to regularise and accelerate an inversion method using analytical streamline sensitivities.

The outline of the paper is as follows: Section 2 presents a simplified model for flow along streamlines underlying the original method of Wang and Kovscek (2000), which is presented in Section 3, and our alternative method, which is presented in Section 4. The two methods are compared in Section 5 using three synthetic examples. Finally, in Section 6 we present the multiscale inversion method and make a few comparisons.

# 2 Basic Flow Model

The original method of Wang and Kovscek (2000) and the alternative inversion method we present in Section 4 are both derived from the same simplified flow model for incompressible flow of two fluids (oil and water) in a single horizontal rock layer. For simplicity we assume piston-like displacement with no capillary forces, where the injected water displaces the in-situ oil instantaneously at the water front. The front position is located at x and the total length of the streamline is L. The pressure drops behind and ahead of the front are denoted  $\Delta p_o$  and  $\Delta p_w$ , respectively. Finally, we assume (for the moment) that gravity can be neglected.

The average Darcy velocities for oil and water are given by

$$u_o = -k\lambda_o \frac{\Delta p_o}{L-x}, u_w = -k\lambda_w \frac{\Delta p_w}{x},\tag{1}$$

where k represents the effective permeability and  $\lambda_o$  and  $\lambda_w$  the oil- and water end-point mobilities. The total pressure drop over the flow region is given by

$$\Delta p = \Delta p_w + \Delta p_o. \tag{2}$$

For an incompressible system, the two velocities are equal, i.e.,

$$u = u_o = u_w.$$

The actual front velocity v, derived by mass conservation over the front, is given by

$$v = \frac{dx}{dt} = \frac{u}{\Delta S \cdot \phi},\tag{3}$$

where  $\Delta S = 1 - S_{or} - S_{wir}$  is the difference in end-point saturation and  $\phi$  is the effective porosity.

Combining the above equations, we wind up with the following ordinary differential equation for the front position

$$\frac{dx}{dt} = -\frac{\Delta p}{\phi \Delta S} \frac{k}{\frac{x}{\lambda_w} + \frac{L-x}{\lambda_o}} = -\frac{\lambda_w \Delta p}{\phi \Delta S} \frac{k}{ML + (1-M)x},\tag{4}$$

where  $M = \lambda_w / \lambda_o$  is the end-point mobility ratio.

The effective streamline permeabilities are calculated based on the permeabilities of the underlying simulation grid. More precisely, the effective permeability of a streamline is given by the harmonic average, weighted by the time-of-flight through grid blocks,

$$k = \frac{\sum_j \tau_j}{\sum_j \frac{\tau_j}{K_j}}, \quad j \in N_b.$$

Here  $N_b$  is the set of indices of the grid blocks the streamline intersects,  $K_j$  is the permeability of grid block j, and  $\tau_j$  is the associated increment in time-of-flight.

Because we regard an incompressible system, each streamline can only originate from an injection well and terminate at a production well. In other words, each streamline will connect an injector to a producer. If  $q_i$  denotes the flow rate of streamline *i*, the total flow rate of a well with *N* streamlines connected is given by

$$q = \sum_{i=1}^{N} q_i.$$

In the next section we will derive the Wang–Kovscek inversion method, assuming that all streamlines connecting an injector and a producer can be modelled as a set of non-communicating (horizontal) layers, where the flow in each layer is described as outlined above.

# 3 The Wang–Kovscek Method

Wang and Kovscek (2000) introduced an iterative method for modifying permeabilities defined on a grid to match calculated with observed production data. The method is based upon two ideas: (i) the flow in an injector-producer pair can be represented by a set of N streamlines, where each streamline contributes a given amount (rate, pressure drop) to the production data; and (ii) each increment in the fractional-flow curve of the producer can be associated with the breakthrough of the injected fluid in a single streamline. By aligning the streamlines according to breakthrough times, one can identify the streamline causing a certain increment in the production data. Assuming that the streamlines in the estimated model is approximately the same as in the true model, the effective properties (here permeability) of the streamline can then be adjusted to match the corresponding increment in the observed fractional flow.

Each iteration of the Wang–Kovscek method requires one flow simulation and consists of two steps: First, modifications of the effective streamline permeabilities are identified in accordance with the mismatch in fractional-flow, well pressures, and well rates. Second, the perturbed effective permeabilities are propagated to the underlying grid in physical space and a flow simulation is performed to check the match. Although we here only present the method for a single injector-producer pair, the extension to multiple wells is straightforward.

Next we will outline how to obtain the permeability modifications for fractional flow and for pressure drop and flow rate.

# 3.1 Match of Fractional-Flow

The fractional-flow at a producer is not matched directly, but is used to align observed and calculated breakthrough times for the streamlines terminating at the producer. By assuming that the order of breakthroughs is approximately the same for the estimated and the true permeability fields, the mismatches in breakthrough times for the streamlines are then applied to modify the streamline permeability.

If we assume piston-like displacement and apply a streamline formulation with



Fig. 1. Observed and calculated fractional-flow curves are used to obtain breakthrough times.

equal total flow-rate for all streamlines, each streamline will contribute equally to the total fractional-flow curve at the producer. Observed and calculated breakthrough times can therefore easily be obtained from the corresponding fractional-flow curves, see Figure 1. The assumption of equal flow rate for the streamlines is a simplifying assumption and is not crucial for the derivation of the method. However, if the streamlines have different flow rates, each streamline will contribute differently to the fractional-flow curve when breaking through, and the order in which the streamlines break through may be more important.

To derive an expression for the breakthrough times in each of the N individual streamlines connecting an injector to a producer, Wang and Kovscek models the flow along N streamlines as the flow in N non-communicating (horizontal) layers. This means changing perspective from streamlines to streamtubes, such that each "streamline" is assigned a certain flow volume. Suppose now that each layer has length  $L_i$ , average cross-sectional area  $A_i$ , average porosity  $\phi_i$ and permeability  $k_i$ , and end-point saturation difference  $\Delta S_i = 1 - S_{wir} - S_{or}$ . Expressed in terms of *pore-volumes injected* (PVI) for the injector-producer pair, the breakthrough time  $\tilde{T}_i$  of streamline *i* can now be written

$$\widetilde{T}_i = \frac{\sum_{j=1}^N (A\phi L)_j \widetilde{x}_j^i}{\sum_{j=1}^N (A\phi L)_j}.$$
(5)

Here  $\tilde{x}_j^i$  represents the relative front position along streamline j when streamline i breaks through, and is provided by Dykstra–Parsons' method (Dykstra and Parsons, 1950). The relative front position can be derived from the differential equation (4) for the front position in a single layer. Dividing the expressions for layers j and i, we obtain

$$\frac{d\tilde{x}_j}{d\tilde{x}_i} = F_j^i \frac{M_i + (1 - M_i)\tilde{x}_i}{M_j + (1 - M_j)\tilde{x}_j}, \qquad F_j^i = \frac{k_j \phi_i \Delta S_i \lambda_{wj} L_i^2}{k_i \phi_j \Delta S_j \lambda_{wi} L_j^2}, \qquad \tilde{x} = \frac{x}{L}.$$

Integrating this differential equation over the streamlines and evaluating  $\tilde{x}_j$  at breakthrough for streamline i ( $\tilde{x}_i = 1$ ) gives  $\tilde{x}_j^i$ .

The next step is to use this simplified flow model to relate the discrepancies in breakthrough times to discrepancies in effective streamline permeabilities  $k_i$ . Because the sum in the numerator of (5) runs over all streamlines, the breakthrough time  $\tilde{T}_i$  is a function of the permeabilities of all streamlines. Linear approximation therefore gives

$$\Delta \widetilde{T}_{i} = \frac{\partial \widetilde{T}_{i}}{\partial k_{1}} \Delta k_{1} + \frac{\partial \widetilde{T}_{i}}{\partial k_{2}} \Delta k_{2} + \frac{\partial \widetilde{T}_{i}}{\partial k_{3}} \Delta k_{3} + \ldots + \frac{\partial \widetilde{T}_{i}}{\partial k_{n}} \Delta k_{n},$$
(6)

where

$$\Delta k_i = k_i^{obs} - k_i^{cal}$$
 and  $\Delta \widetilde{T}_i = \widetilde{T}_i^{obs} - \widetilde{T}_i^{cal}$ .

This equation gives a relation between the mismatch in breakthrough times for streamline *i* and the permeability modifications for all streamlines. The derivatives  $a_{ij} = \partial \tilde{T}_i / \partial k_j$  are obtained by differentiating (5). Applying the same linear approximation for all streamlines results in the following system

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} \Delta k_1 \\ \Delta k_2 \\ \vdots \\ \Delta k_N \end{bmatrix} = \begin{bmatrix} \Delta \tilde{T}_1 \\ \Delta \tilde{T}_2 \\ \vdots \\ \Delta \tilde{T}_N \end{bmatrix}.$$
(7)

The system is simplified by defining relative or normal parameters (Wang and Kovscek, 2000). For unit mobility ratios, the system is strongly diagonally dominant and approximately decouples so that

$$\frac{\Delta k_i^t}{k_i^{cal}} \approx \frac{\tilde{T}_i^{cal} - \tilde{T}_i^{obs}}{\tilde{T}_i^{obs}}.$$
(8)

The superscript t indicates that the modification is due to mismatch in breakthrough time. As explained by Wang and Kovscek (2000), the approximation (8) becomes better the more streamlines are involved, because off-diagonal elements scale like 1/N. The approximation (8) is used for practical applications (even for nonunit mobility ratios) to obtain the modifications; see Wang and Kovscek (2000); Caers et al. (2002); Caers (2003).

#### 3.2 Match of Pressure Drop and Rate

The description in Wang and Kovscek (2000) is a bit ambiguous as to how the modifications due to pressure drop and rate are calculated. In this section we describe how we understood the derivation of pressure and rate modifications. Further, these derivations create a foundation for parts of the derivation of our improved method.

If we assume that all streamlines have the same flow rate, the error in flow rate of an injector-producer pair is distributed equally among all streamlines. If a streamline formulation with varying streamline rate is used, the error in flow rate may instead be distributed by weighting. The error in pressure drop is common to all streamlines of an injector-producer pair.

To modify the permeability due to mismatch in pressure drop and rate, an expression relating these three quantities is used. To derive this expression we start out by (4), reading

$$v_i = -\frac{\lambda_w \,\Delta p}{\phi_i \,\Delta S} \,\frac{k_i}{ML_i + (1-M)x_i}$$

We regard  $\Delta p$  as the effective pressure drop for the streamline, possibly obtained by temporal averaging over the depletion period. Averaging the velocity over the streamline then gives

$$\bar{v}_i = -\frac{\ln M}{M-1} \frac{k_i \lambda_w \,\Delta p}{L_i \,\phi_i \,\Delta S}.\tag{9}$$

The average actual front velocity can be estimated by

$$\bar{v}_i = \frac{q_i}{\phi_i \Delta S A_i}.\tag{10}$$

Here  $A_i$  is the average cross-sectional area of a streamline/streamtube and  $q_i$  is the effective streamline rate derived by distributing the well rate among all streamlines connected to a well. Having an explicit expression for  $A_i$  is not important, since it will cancel out later in the derivation. Finally, rearranging (9) gives the permeability by

$$k_i = -\frac{M-1}{\ln M} \frac{\bar{v}_i L_i \phi_i \Delta S}{\lambda_w \Delta p}.$$
(11)

In the limit of unit mobility ratio, the last expression can be obtained directly from the averaged Darcy's law (1).

Evaluating (11) for calculated and observed data, with  $\bar{v}_i$  estimated by (10), we obtain

$$\frac{\Delta k_i^{p,q}}{k_i^{cal}} = \frac{\Delta p^{cal} \, q_i^{obs} - \Delta p^{obs} \, q_i^{cal}}{\Delta p^{obs} \, q_i^{cal}}.$$
(12)

The superscripts p and q indicate that the corresponding modification is due to mismatch in pressure drop and rate. If there are no observation for the pressure drop or the rate, it would be natural to assume that the calculated response for the quantity is correct, i.e., calculated and observed responses coincide. This will make the quantity cancel out from the expression for the modification.

# 3.3 Updating Grid Permeability

To obtain a total correction factor  $r_i$  for streamline *i*, geometric averaging is used to combine the relative modifications (8) and (12):

$$r_i = \left[ \left( 1 + \Delta k_i^t / k_i^{cal} \right) \cdot \left( 1 + \Delta k_i^{p,q} / k_i^{cal} \right) \right]^{1/2}.$$

If the streamline distribution is updated during the forward simulation, one or several of the temporary streamline distributions are used for the inversion; see the discussion in Section 4.4.

For simplicity, and to focus on the improved quality of the streamline permeability corrections, we will in the following only use a simple deterministic method to propagate the modifications for the streamline permeabilities onto the underlying geological grid; see Wang and Kovscek (2000). More sophisticated geostatistical mapping methods incorporating e.g., prior information on the permeability distribution have been developed by Caers et al. (2002); Caers (2003); Caers et al. (2004). In Caers et al. (2002) the grid permeabilities are described by a Gauss-Markov random function. The grid permeabilities are modified by sampling from the random function conditioning on the updated streamline effective permeabilities  $k_i^{new}$ . This approach honors the variogram and the histogram and may therefore result in permeability fields that better preserve geologic realism. In Caers (2003) the mapping is performed using gradual deformation, which preserve the geological continuity. Further, the use of multiple-point geostatistics may enable history matching of complex heterogeneous geological structures, like fractures and channels, which is beyond the scope of variogram-based methods. Finally, in Caers et al. (2004) the effective permeability modifications are obtained for the flow zone associated with each producer. The mapping to the grid is performed with Direct Sequential Simulation (DSSIM) to honor the variogram while allowing the histogram to change.

# 4 Improved Inversion of Effective Streamline Permeabilities

In this section we present an alternative inversion method that is not based upon the Dykstra–Parson algorithm. Instead, our method solves the differential equation (4) for the front position in each streamline and uses these solutions to relate the discrepancies in observed and calculated breakthrough times to perturbations of the effective streamline permeabilities. There are several advantages to this approach: (i) we avoid solving the linear system (7) or making further approximations by decoupling to (8); (ii) our new formulation is easier to generalise; and (iii) we generally obtain faster convergence and a better history match.

In the Dykstra–Parsons method, (4) is used to relate positions of saturation fronts in different streamlines. This eliminates real time and pressure, and couples the streamlines. We propose to avoid this coupling by using (4) directly to model absolute front positions in real time for each streamline. Integrating (4) over streamline i

$$\int_{0}^{L_{i}} [ML_{i} + (1 - M)x_{i}] dx_{i} = -\int_{0}^{T_{i}} \frac{k_{i} \lambda_{w} \Delta p(t)}{\phi_{i} \Delta S} dt$$
$$\frac{1}{2} (M + 1) L_{i}^{2} = -\frac{k_{i} \lambda_{w}}{\phi_{i} \Delta S} \int_{0}^{T_{i}} \Delta p(t) dt,$$

and solving for the effective permeability  $k_i$  gives

$$k_i = -\frac{M+1}{2} \frac{L_i^2 \phi_i \Delta S}{\lambda_w \int_0^{T_i} \Delta p(t) dt} = -\frac{M+1}{2} \frac{L_i^2 \phi_i \Delta S}{\lambda_w} \frac{1}{\overline{\Delta p} T_i}.$$
 (13)

Here  $\overline{\Delta p}$  is the temporal average of the pressure drop over  $[0, T_i]$ .

Evaluating the permeability for observed and calculated data gives the relative modification

$$\frac{\Delta k_i^{t,p}}{k_i} = \frac{k_i^{obs} - k_i^{cal}}{k_i^{cal}} = \frac{\overline{\Delta p}^{cal} T_i^{cal} - \overline{\Delta p}^{obs} T_i^{obs}}{\overline{\Delta p}^{obs} T_i^{obs}},$$
(14)

for streamline *i*. If  $\bar{v}_i$  is estimated by

$$\bar{v}_i = L_i / T_i,\tag{15}$$

the permeability modifications (13) and (11) only differ by the mobility factors  $(M-1)/\ln M$  and (M+1)/2. However, for unit mobility ratio the two expressions coincide.

Notice that no mathematical approximations were made in the derivation of (14) from (4). Under the same physical assumptions, the modifications (14) should therefore be more accurate than those obtained by (7) and (8). Further, notice that we also could have solved for  $\phi_i$ , or for  $(k_i/\phi_i)$ , instead of  $k_i$  in (13), and thereby obtained expressions for relative modifications in effective porosity or permeability-porosity ratios.

#### 4.1 Accounting for Gravity along Streamlines

Using the new inversion method introduced above, it is straightforward to account for gravity in the flow direction. To this end, consider the Darcy velocities for oil and water in the presence of gravity

$$u_{oi} = -k_i \lambda_o \left( \frac{\Delta p_o}{L_i - x_i} + \rho_o g \sin \alpha_i \right)$$
  

$$u_{wi} = -k_i \lambda_w \left( \frac{\Delta p_w}{x_i} + \rho_w g \sin \alpha_i \right).$$
(16)

Here  $\alpha_i$  is the streamline effective dip angle,  $g = |\vec{g}|$  the acceleration of gravity, and  $\rho_{\alpha}$  the density of phase  $\alpha$ . The effective dip angle of a streamline  $\Psi_i$  is given by the time-of-flight weighted average of the local dip angle  $\alpha(\tau)$ ,

$$\alpha_i = \frac{\int_{\Psi_i} \alpha(\tau) d\tau}{\int_{\Psi_i} d\tau}.$$

Combining (2) and (3) with (16), we can now extend the differential equation (4) for the front position to account for gravity

$$\frac{dx_i}{dt} = -\frac{k_i \lambda_w}{\phi_i \Delta S} \cdot \frac{[\Delta p + \rho_o g L_i \sin \alpha_i + x_i (\rho_w - \rho_o) g \sin \alpha_i]}{ML_i + (1 - M)x_i}.$$
(17)

For constant pressure drop, the same procedure as above gives

$$\frac{\Delta k_i}{k_i} = \frac{T_i^{cal} I^{obs} - T_i^{obs} I^{cal}}{T_i^{obs} I^{cal}},\tag{18}$$

where

$$I^{\xi} = \int_{0}^{L_{i}} \frac{M L_{i} + (1 - M) x_{i}}{\Delta p^{\xi} + \rho_{o} g L_{i} \sin \alpha_{i} + (\rho_{w} - \rho_{o}) g x_{i} \sin \alpha_{i}} dx_{i}, \quad \xi = cal, obs.$$
(19)

The integral has the following analytic solution

$$I^{\xi} = c^{-2} \left( a \, c - b \, d \right) \cdot \ln(|d + c \, L_i| / |d|) + b \, c^{-1} \, L_i,$$

where  $a = L_i M$ , b = 1 - M,  $c = (\rho_w - \rho_o) g \sin \alpha_i$ , and  $d = \Delta p^{\xi} + \rho_o g L_i \sin \alpha_i$ . For unit mobility ratios the integral simplifies considerably. Letting the effective dip angle turn to zero in (19), (18) simplifies to (14).

#### Time-Dependent Pressure Drops

Allowing for a time-dependent pressure drop in (17) and rearranging we obtain the following first-order differential equation

$$e(a+bx_i)\dot{x}_i + k_i cx_i = -k_i d(t),$$
(20)

where  $e = (\phi_i \cdot \Delta S / \lambda_w)$ , while *a*, *b*, *c* and *d* are as given above, except that the pressure drop in *d* is now time-dependent. This equation generally has a nonlinearity in the first term. However, for unit mobility ratios it is linear and can be solved for the special case of  $\rho_o = \rho_w$ 

$$k_i = -\frac{L_i^2 \phi_i \Delta S}{\lambda} \frac{1}{\left(\overline{\Delta p} + \rho g L_i \sin \alpha_i\right) T_i}.$$
(21)

In the limit  $\alpha_i \to 0$ , (21) simplifies to (13).

In the general case we will proceed as in Section 3.2 and rely on spatial averaging rather than trying to solve (20) explicitly for  $k_i$ . The front velocity  $v_i = dx_i/dt$  is given by (17), where we regard  $\Delta p$  as effective pressure drop for the streamline, possibly obtained by temporal averaging over the depletion period. Spatial averaging of the front velocity over the streamline gives

$$\bar{v}_i = -\frac{k_i \lambda_w}{L_i \phi_i \Delta S} \frac{1}{(M-1)^2} \cdot \left(a(M)\Delta p - (b(M)\rho_o - c(M)\rho_w) g L_i \sin \alpha_i\right),$$

where  $a(M) = (M - 1) \ln M$ ,  $b(M) = \ln M - M + 1$ , and  $c(M) = M \ln M - M + 1$ . For unit mobility ratios the average front velocity is

$$\bar{v}_i = -\frac{k_i \lambda_w}{L_i \phi_i \Delta S} \Big( \Delta p + \frac{1}{2} (\rho_o + \rho_w) g L_i \sin \alpha_i \Big).$$

Similarly to (11), we wind up with

$$k_i = -\frac{\bar{v}_i L_i \phi_i \Delta S (M-1)^2}{\lambda_w \left[a(M)\Delta p - (b(M)\rho_o - c(M)\rho_w) g L_i \sin \alpha_i\right]}$$
(22)

for nonunit mobility ratio, while for unit mobility ratio we get

$$k_i = -\frac{\bar{v}_i L_i \phi_i \Delta S}{\lambda_w (\Delta p + \frac{1}{2}(\rho_o + \rho_w) g L_i \sin \alpha_i)}.$$
(23)

These two expressions, with  $\bar{v}_i$  estimated by (10) or (15), can be used to obtain streamline modifications. An advantage of these expressions is that the streamline rate can be included. As expected, letting  $\alpha_i$  tend to zero in (22) and (23) results in (11). For the case with constant pressure drop, equal densities  $\rho_o = \rho_w$ , and  $\bar{v}_i$  estimated by (15), one can show that the modification (23) coincides with the exact modification (21).

#### 4.2 Match of Rate

For several injectors and/or producers, it may be necessary to match the error in the total flow rate of the wells considered. The rate can be matched similarly to how it is done for the Wang–Kovscek method. Combining the permeability expressions (11), (22) and (23) with (10), relates the permeability and the streamline rate. By using these permeability expressions, relative modifications  $\Delta k_i^{p,q}/k_i^{cal}$  for rate and pressure can be obtained (see (12)).

# 4.3 Total Modifications

If breakthrough times and pressure drops are matched, the correction factor becomes

$$r_i^{t,p} = 1 + \Delta k_i^{t,p} / k_i^{cal}$$

This modification factor can be combined with the modification due to rate and pressure drop  $\Delta k_i^{p,q}/k_i^{cal}$  to give a total correction factor by the geometric average (possibly weighted)

$$r_i = \left[ \left( 1 + \Delta k_i^{t,p} / k_i^{cal} \right) \cdot \left( 1 + \Delta k_i^{p,q} / k_i^{cal} \right) \right]^{1/2}.$$

Since the mismatch in pressure drop contributes to both the relative modifications, the mismatch in pressure drop may be distributed between the two expressions. The modification factors can be propagated to the underlying simulation grid by any of the geostatistical approaches described in Section 3.3. However, we will for our implementations use the simple deterministic approach proposed in Wang and Kovscek (2000). The breakthrough times are obtained by the approach described in Section 3.1.

# 4.4 Evolving Streamlines

A general problem for streamline-based history matching is the fact that the streamlines only exists during a single pressure time step. If streamlines are evolving, one should ideally compose effective streamlines by adding one segment for each time step. This requires that one is able to keep track of each streamline from one time step to the next. This means that the number of streamlines for each injector-producer pair must be constant, which is a hard constraint to fulfil for a general streamline implementation. To deal with the problem of streamlines existing for a single pressure step, Wang and Kovscek (2000) suggest to pick one of the temporary streamline distributions as a representative streamline distribution. The geometry and time-of-flight information is therefore not correct for the whole streamline, so the calculations of effective properties may be inaccurate. However, this is not critical, because effective properties are only used to *perturb* the history match in a given direction and the quality of this perturbation is estimated in the consecutive forward flow simulation.

# 5 Numerical Examples

In this section we assess the new inversion methods introduced in the previous section. To make the comparison with the original Wang–Kovseck method as clean as possible, we focus on simple and idealised test cases with a small number of parameters. Applications of the Wang–Kovseck method to more realistic test cases and real reservoirs can be found in Gross et al. (2004); Caers et al. (2004).

We consider three synthetic reservoirs with dimensions  $200 \times 200 \times 10 \text{ m}^3$ ,



Fig. 2. The true permeability field in mD for Cases 1 and 2 (left) and Case 3 (right). where pure water ( $\rho_w = 1000 \text{ kg/m}^3$ ,  $\mu_w = 1 \text{ cp}$ ) is injected at a rate of 300 STB/day into a reservoir initially filled with pure oil ( $\rho_o = 700 \text{ kg/m}^3$ ,  $\mu_o = 1 \text{ cp}$ ). Further, we assume a zero residual oil saturation after depletion.

Case 1: Quarter Five-Spot

We first consider a quarter five-spot example with no flow over the outer boundaries and one injector in the lower-left corner and a producer in the upper-right corner. Because the total flow rate at the wells is preserved, only data from fractional flow and pressure drop are matched for this example. For the flow model we assume linear relative permeabilities,  $\lambda_w(s) = s$  and  $\lambda_o(s) = 1 - s$ .

The true permeability is represented on a  $20 \times 20 \times 1$  uniform grid and consists of a low-permeable background with an ellipsoidal high-permeability region imposed along the diagonal; see Figure 2. Matching the main permeability trends should be rather easy for any streamline-based method, since the highpermeable region is aligned with the major flow direction (from injector to producer). For both methods we start the iterations from a homogeneous permeability field of 150 mD (no prior information).

Figure 3 shows inferred permeability fields, matched fractional flow curves, and relative errors obtained using nine iterations with the original Wang–Kovscek inversion method, (8) and (12), and with our improved method, (14). Both methods match the fractional flow and pressure drop, but whereas the Wang– Kovscek method has not converged fully after nine iterations, our method has converged after five. The inferred permeability fields for the two methods are qualitatively similar; both methods capture the high-permeable region, but the estimated permeability is too low in both the ellipsoidal region and in the low-permeable background. On the other hand, the permeabilities close to the wells are too high, which makes the effective streamline permeabilities consistent with the effective permeabilities of the true permeability field.



Fig. 3. Case 1. Comparison of inferred permeability fields, matched fractional-flow curves, and relative errors for the original Wang–Kovseck method (left) and our new method (right). In the middle plot, the observed fractional flow is given by a solid line, the calculated curve by a dotted line, and matched curves by dashed lines.



Fig. 4. Case 1. Matched fractional-flow curves (dashed lines) with five percent white noise added to the observed fractional-flow curve (solid line).



Fig. 5. Case 2. Comparison of inferred permeability fields and matched fractional-flow curves with and without accounting for gravity (left and right, respectively).

If five percent white noise is added to the observed fractional-flow curve, our method is still able to match the fractional flow (see Figure 4) if we avoid using the streamlines contributing to approximately the upper five percent of the fractional-flow curve. Due to the flatness in this part of the curve, the calculated breakthrough times are more sensitive to noise.

# Case 2: Tilted Quarter Five-Spot

We now tilt the reservoir from Case 1, such that the edges of the reservoir are aligned with the vectors [0.9539, 0, 0.3] and [-0.0943, 0.9493, 0.3]. We compare the inversion obtained using two different formulas from Section 4: (18) accounts for gravity along streamlines, and (14) does not.

Figure 5 shows inferred permeability fields and matched fractional flow curves for six iterations. By accounting for gravity we observe a bit faster match for the pressure drops, but apart from this, there is little difference in the matching process. We obtain essentially the same result as for Case 1, both with and without accounting for gravity in the inversion. The reason is that we calculate relative modifications, for which proportionality errors cancel out. Although gravity does not give a pure proportionality error, it has the same impact on calculating both  $I^{cal}$  and  $I^{obs}$  for each streamline; see (18). This



Fig. 6. Case 3. Comparison of inferred permeability fields and relative errors for the original Wang–Kovseck method (left) and our new method (right).

example demonstrates the robustness of using relative modifications.

#### Case 3: Five-Spot with Nonunit Mobility Ratio

Finally, we consider a five-spot pattern in a square domain, but now with an injector in the centre and one producer in each of the four corners. The reference permeability is given on a  $32 \times 32$  grid as shown in Figure 2. We assume quadratic relative permeabilities,  $\lambda_w(s) = s^2/\mu_w$  and  $\lambda_o(s) = (1 - s)^2/\mu_o$  and  $\mu_o = 0.4$  cp, which gives a nonunit end-point mobility ratio of M = 0.4. As in the previous example, we match fractional flows and pressure drops using the original Wang–Kovscek method and our improved method from Section 4. We start the iterations by a homogeneous initial permeability field of 700 mD (no prior information), and five percent white noise is added to the observed fractional-flow curve.

Figure 6 shows the inferred permeability fields and relative errors for the two methods. The error bars show the sum of the relative errors for fractional flow and pressure drop for all producers. As above, our method converges faster, especially for the pressure drop. We observe that both methods capture the mean permeability correctly in the four injector-producer sectors, but fail to capture permeability structures within each sector properly. In particular we notice the artifacts along streamline paths, caused by the direct mapping of



Fig. 7. Illustration of the hierarchically coarsened grids used in the multiscale inversion process.

effective properties and the lack of a prior (geostatistical) model. In the next section we propose a multiscale approach that will improve the resolution of large-scale permeability structures that affect several injector-producer pairs.

# 6 A Multiscale Method

Many inversion methods are based upon minimisation of an objective functional using a gradient descent method. Inverse problems are generally underdetermined in the sense that one has a few observations and a large number of unknown parameters. Moreover, since the inversion process is highly nonlinear, the objective functionals tend to have a large number of local minima that must be avoided. Multiscale inversion has been suggested by several authors as a means to stabilise the inversion and avoid local minima; see e.g., Yoon et al. (1999) for a multiscale inversion method based on analytical streamline sensitivities.

Although our streamline approach is not based upon minimisation of an error functional, we here suggest to use a similar approach to speed up and stabilise the inversion. To this end we introduce a family of hierarchically coarsened grids, as illustrated in Figure 7, where the finest grid coincides with the geogrid on which we seek to match permeabilities. Given the family of grids, the idea is quite simple: Starting with a small set of streamlines, we modify the effective streamline permeabilities to match observed production data as described in Section 4 and map the perturbed streamline permeabilities back onto the coarsest grid. Depending on the complexity of the reservoir, one or more iterations can be performed. The resulting permeabilities on the coarsest grid are then interpolated (linearly) onto the next grid in the family and used as initial values for the match on the next scale. The process is continued until the finest grid is reached, or can be terminated when a sufficiently good match is obtained or if no improvement in the misfit is observed from one level to the next. By allowing for early termination, the resolution of the resulting permeability field will correspond to the information content in the production data, and spurious effects from over-parameterisation are reduced. On the coarser grid levels, the ratio between the number of grid permeabilities to be history-matched and the number of streamlines/data-points can be more favourable, and therefore the inversion problem may be less underdetermined. Even though we modify the permeability on different coarse grids, the streamlines can be traced and fluid simulation can be performed on a much finer grid (e.g., the underlying geogrid) to avoid problems with loss of accuracy and representation of wells.

We will present two simple synthetic cases to illustrate the multiscale approach. For simplicity, we assume that the reservoir is square, start with a uniform  $4 \times 4$  grid as the coarsest grid, and recursively refine by subdividing each grid block into four square grid blocks; see Figure 7. The dimensions of the grids thus become  $4 \times 4$ ,  $8 \times 8$ ,  $16 \times 16$ ,  $32 \times 32$ , etc. In the inversion, we perform only one iteration on all grids, except for the finest grid, where we iterate until convergence. More iterations could have been performed on each refinement level, but we only want to capture the trends of the large-scale permeability structures on each level. The reservoir and fluid parameters (except permeability and the mobility ratio) used in these examples are as described in Section 5. For the two examples we assume quadratic relative permeabilities like for Case 3 and add five percent white noise to the fractional-flow observations.

# Case 4: Quarter Five-Spot

This example is similar to Case 1 from Section 5, except that the true permeability field is represented on a  $32 \times 32$  grid (see Figure 9). Besides, for this example the oil viscosity is 2.5 cp, which gives an end-point mobility ratio of M = 2.5.

Figure 8 shows the inferred permeability field on each refinement level compared with the permeability field inferred after seven iterations of the inversion algorithm directly on the  $32 \times 32$  grid. The high permeable region is located already on the  $4 \times 4$  grid. Even though the inferred permeability field of the multiscale approach seems smoother, the location of the high-permeable region is more accurately positioned and is less smeared out between the wells. On the plot for the final match on the fine grid, the streamline pattern is clearly visible. This effect appears during the last few iterations and seems to be a consequence of an over-parametrisation that could have been avoided if we had terminated the iteration earlier.

Figure 8 also shows the error reduction with and without the multiscale approach. Although the error reduction is slightly slower for the multiscale approach, the same number of iterations are necessary for convergence for both approaches, and therefore the computational effort for the multiscale method



Fig. 8. Case4. Comparison of inferred permeability fields and relative errors with and without the multiscale approach. The true permeability field in mD (upper left). The four next plots (from left to right) show the inferred permeability fields on the  $4 \times 4$ ,  $8 \times 8$ ,  $16 \times 16$ , and  $32 \times 32$  grids. The last plot in the second row shows the inferred permeability field obtained by a direct inversion on the  $32 \times 32$  grid. The two plots on the last row shows the relative error with (left) and without (right) the multiscale approach.

is smaller since fewer iterations are performed on the finest scale.

Case 5: Five-Spot

In the next example we revisit Case 3. Figure 9 shows the inferred permeability field on each refinement level and the inferred field after five iterations directly on the finest grid. Already on the  $4 \times 4$  grid some of the large-scale structures of the permeability field are located. Compared with the inversion in Case 3, the multiscale approach captures more of the large-scale structures in the reference permeability field and avoids the artificial streamline-induced zonation structure observed in Figure 6. Moreover, the multiscale approach is faster, because fewer iterations are necessary on the fine scale.



Fig. 9. Case 5. Comparison of inferred permeability fields and relative errors with and without the multiscale approach. The true permeability field in mD (upper left). The four next plots (from left to right) show the inferred permeability field on the  $4 \times 4$ ,  $8 \times 8$ ,  $16 \times 16$ , and  $32 \times 32$  grids. The last plot in the second row shows the relative errors for the multiscale approach.

# 7 Concluding Remarks

We have suggested two improvements to the streamline inversion method introduced by Wang and Kovscek (2000). The resulting inversion method is able to match production data and capture large-scale permeability structures, but fails to incorporate the (a priori) variability of the permeability field. Combining the inversion method with existing geostatistical inversion methods, the method can be extended to yield inferred permeability fields that also satisfy geological constraints.

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