Multiscale Methods and Streamline Simulation for Rapid Reservoir Performance Prediction

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Summary. We introduce a novel multiscale approach for reservoir simulation as an alternative to industry-standard upscaling methods. In our approach, reservoir pressure and total velocity is computed separately from the fluid transport. Pressure is computed on a coarse grid using a multiscale mixed-finite element method that gives a mass-conserving velocities on a fine subgrid. The fluid transport is computed using streamlines on the underlying fine geogrid.

Key words: multiscale methods, porous media, upscaling, streamlines

1 Introduction

The size of geomodels used for reservoir description typically exceeds by several orders of magnitude the capabilities of conventional reservoir simulators based upon finite differences. These simulators therefore employ upscaling techniques that construct coarsened reservoir models with a reduced set of geophysical parameters. In this way the size of the simulation model is reduced so that simulations can run within an acceptable time-frame.

Streamline methods are gaining in popularity and have a potential of simulating much larger reservoir models than what is possible using traditional finite difference simulators. Streamline methods are based upon a fractional flow formulation, where the model is split into an elliptic/parabolic pressure equation and hyperbolic fluid transport equations. For immiscible, incompressible fluids and negligible gravity and capillary forces, the equations read

$$\nabla \cdot v = q, \qquad v = -K\lambda_t \nabla p, \tag{1}$$

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot f_w(S)v = q_w. \tag{2}$$

Here p denotes pressure, v the total velocity, S water saturation, K rock permeability, $\lambda_t(S)$ total mobility, and ϕ rock porosity. The two equations are solved sequentially: first the pressure equation (1) is solved to give a velocity field, by which the saturations can be transported according to (2), and so on.

A major obstacle in applying streamline methods to large geomodels is the need for accurate and efficient solution of the pressure equation (1). In particular, the pressure solver must be locally (and globally) mass-conservative and should handle: (i) irregular grids that conform to geological structures; (ii) strongly heterogeneous and anisotropic formations; and (iii) flows with large dynamic aspect ratios. Mixed finite element methods (MFEM) and multi-point flux-approximation finite-volume methods (MPFA) are examples of methods that handle these properties, and cover the most widely used methods for elliptic problems where mass preservation is an issue.

Here we present a new simulation method for incompressible, immiscible two-phase flow on Cartesian grids. Pressure and velocities are computed using a multiscale, mixed finite-element method (MsFEM) [3, 1], where the pressure is computed on a coarse grid and a mass-conservative velocity field is computed on the underlying fine grid, using numerically constructed base functions with subgrid resolution on the coarse grid. Together with streamline computation of fluid transport, this gives an efficient and robust method that resolves detailed flow patterns on the underlying fine grid. A more detailed study of this multiscale method is presented in [2]. Our main point here is to indicate that the combination of multiscale pressure solvers and streamline methods has a great potential for bridging the gap between high-resolution geomodels and the capabilities of current reservoir simulators.

2 Streamline Method

Streamlines are flow-paths traced out by a particle being passively advected by an external flow field such that the streamline is tangential to the flow velocity at every point. The streamlines can be parametrised by the *time-of-flight* τ, which measures the travel time along each streamline. In our case,

$$v \cdot \nabla\tau = \phi \qquad \text{or equivalently} \qquad \partial\tau = \phi/|v|\, ds. \tag{3}$$

Together with the bistream functions ψ and χ, which satisfy $u = \nabla\psi \times \nabla\chi$, the streamlines define a formal spatial coordinate transform. Applied to the saturation equation (2), for which $u = v/\phi$, this transformation gives

$$S_t + f(S)_\tau = 0. \tag{4}$$

Streamline simulators compute the fluid transport by solving one-dimensional equations like (4) along streamlines in 3D. Here we use a very efficient *front-tracking method* [6] to solve (4). The method starts from piecewise initial data, approximates the flux function by a piecewise linear function, and solves the corresponding Cauchy problem exactly.

3 Multiscale Mixed Finite-Elements

The mixed formulation of (1) over a domain $\Omega \in \mathbb{R}^3$ reads: find $(p, v) \in L^2(\Omega) \times H_0^{1,\mathrm{div}}(\Omega)$ such that

$$\begin{aligned} \iiint_\Omega (K\lambda_t)^{-1} v \cdot u \, dx - \iiint_\Omega p \, \nabla \cdot u \, dx = 0, \\ \iiint_\Omega l \, \nabla \cdot v \, dx = \iiint_\Omega ql \, dx, \end{aligned} \tag{5}$$

for all $u \in H_0^{1,\mathrm{div}}(\Omega)$ and $l \in L^2(\Omega)$. In a mixed-finite element method, the approximation space for v is spanned by a finite set of base functions $\{\psi\} \subset H_0^{1,\mathrm{div}}(\Omega)$; for instance, a set of piecewise linear functions as in the Raviart–Thomas elements of lowest order. In the multiscale method, the base functions are computed numerically by solving a subgrid problem for each interface Γ_{ij} between two coarse grid blocks T_i and T_j

$$\begin{aligned} (\nabla \cdot \psi_{ij})|_{T_i} &= \begin{cases} 1/|T_i|, & \text{if } \int_{T_i} q \, dx = 0, \\ q / \int_{T_i} q \, dx, & \text{otherwise}, \end{cases} \\ (\nabla \cdot \psi_{ij})|_{T_j} &= \begin{cases} -1/|T_j|, & \text{if } \int_{T_j} q \, dx = 0, \\ -q / \int_{T_j} q \, dx, & \text{otherwise} \end{cases} \end{aligned} \tag{6}$$

with no-flow boundary conditions $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$. These numerically generated base functions guarantee a velocity approximation with subgrid resolution. The approximation is mass conservative on the subgrid if the subgrid problems (6) are solved with a mass-conservative method. The base functions ψ_{ij} will generally be time dependent since they depend on λ_t, which is time dependent through $S(x, t)$. For incompressible two-phase flow it is sufficient to regenerate only a small portion of the base functions in each pressure step since the mobility only varies significantly near strong saturation fronts.

4 Numerical Results

To demonstrate that our multiscale method is a viable and robust approach, we present numerical results for Model 2 in the tenth SPE comparative solution project [4]. The model was designed as a benchmark for various upscaling techniques and contains a stack of two heterogeneous formations, see Figure 1. Both formations have large permeability variations, 8–12 orders of magnitude, but are qualitatively different. The Tarbert formation is smooth, and therefore not too hard to upscale. The Upper Ness formation is fluvial with narrow and intertwined flow channels of high permeability.

We compare our simulation results with a reference solution obtained by direct simulation on the fine grid using a standard two-point finite-volume

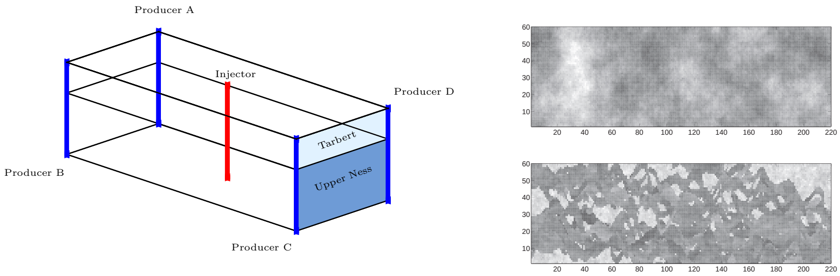

Fig. 1. Schematic of the reservoir model used in [4]. The reservoir dimensions are $1200 \times 2200 \times 170$ ft., and the model consists of $60 \times 220 \times 85$ grid cells. The top and bottom plots to the right depicts the logarithm of the horizontal permeability in the top layer of the Tarbert formation and the bottom layer of the Upper Ness formation.

method. We also compare with the nested gridding method of Gautier et al. [5], which can be considered as the upscaling-based analogue of our method. In the nested-gridding method the absolute mobility ($K\lambda_t$) is upscaled by solving local flow problems. Secondly, the pressure equation is solved on the coarse grid using the upscaled absolute mobilities. Finally, the coarse-grid fluxes are used to determine boundary conditions for local subgrid problems that are solved to obtain a mass-conservative velocity on the subgrid scale. The fluid transport is solved using streamlines for all three methods.

Figure 2 shows a plot of the fraction of water in the produced fluid (water cut) as a function of time for 2000 days of production. The time steps are 25 days up to day 250, 50 days up to day 500, 100 days up to day 1000, and then 200 days. The performance of our multiscale method is remarkably good; the match is almost exact for all four producers and the fine-scale flow channels are reproduced to a large extent as can be seen in Figure 3. Although the nested-gridding method has subgrid resolution, it does not account for the coupling between fine-grid and coarse-grid effects and therefore fails to reproduce the individual water cuts correctly.

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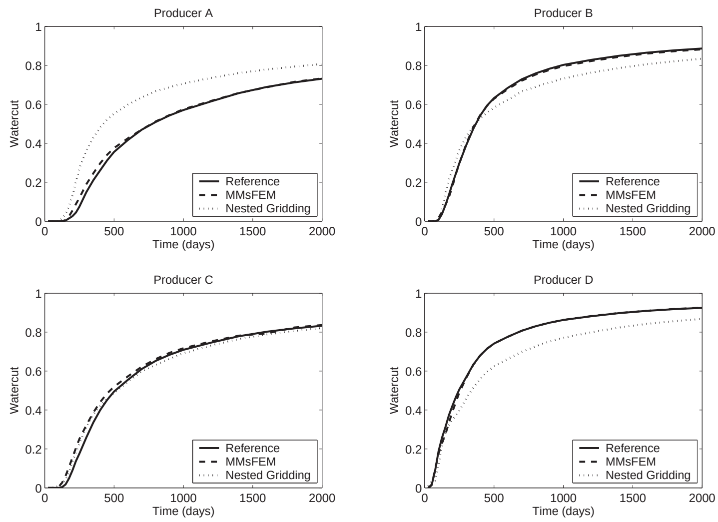

Fig. 2. Water cut curves after 2000 days of simulation.

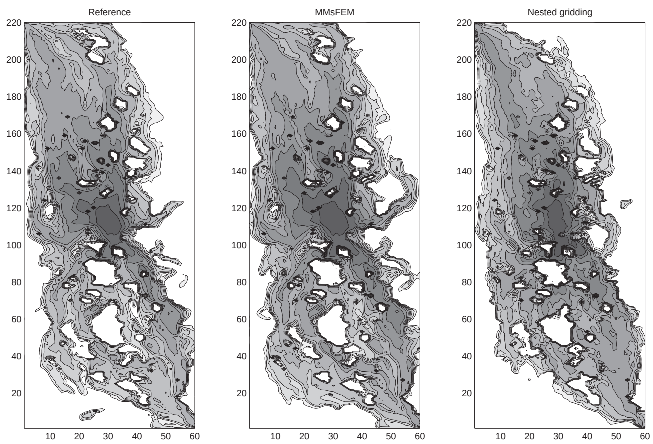

Fig. 3. Water saturation in the bottom layer after 800 days.

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