Introduction to Flow Simulation as Used in Reservoir Engineering

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Outline

Petroleum reservoirs

Naturally occurring flammable liquid/gases found in geological formations

- ▶ Originating from organic sediments that have been compressed and 'cooked' to form hydrocarbons that migrated upward in sedimentary rocks until limited by a trapping structure
- ▶ Found in shallow reservoirs on land and deep under the seabed
- ▶ Only 30% of the reserves are 'conventional'; remaining 70% include *shale oil and gas*, heavy oil, extra heavy oil, and oil sands.
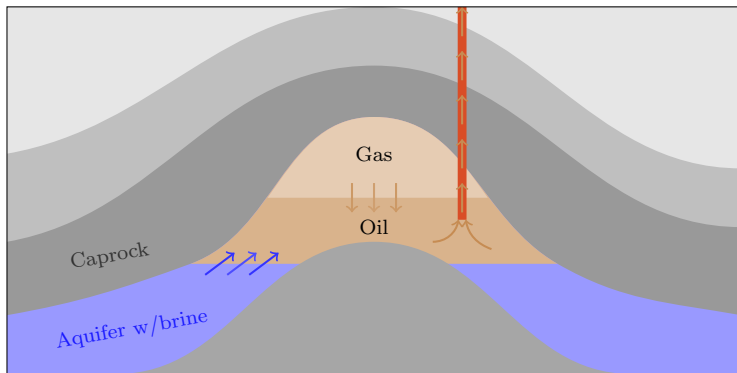
Uses of (refined) petroleum:

- ▶ Fuel (gas, liquid, solid)
- ▶ Alkenes manufactured into plastics and compounds
- ▶ Lubricants, wax, paraffin wax
- ▶ Pesticides and fertilizers for agriculture
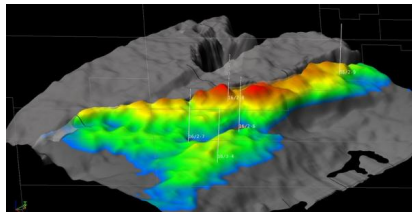
Johan Sverdrup, new Norwegian 'elephant' discovery, 2011.
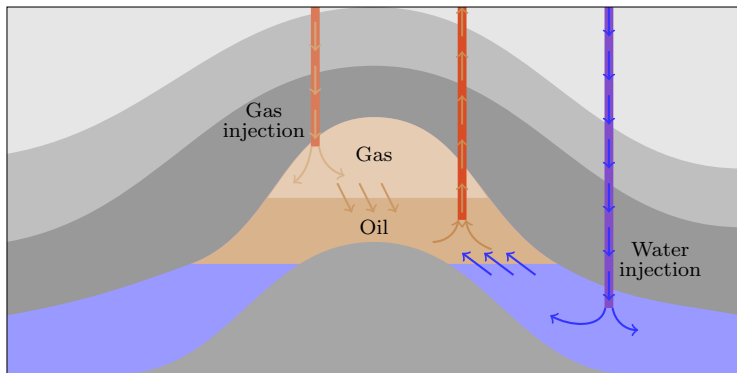Expected to be producing for the next 30+ years

Primary production – puncturing the 'balloon'

When the first well is drilled and opened for production, trapped hydrocarbon starts flowing toward the well because of over-pressure
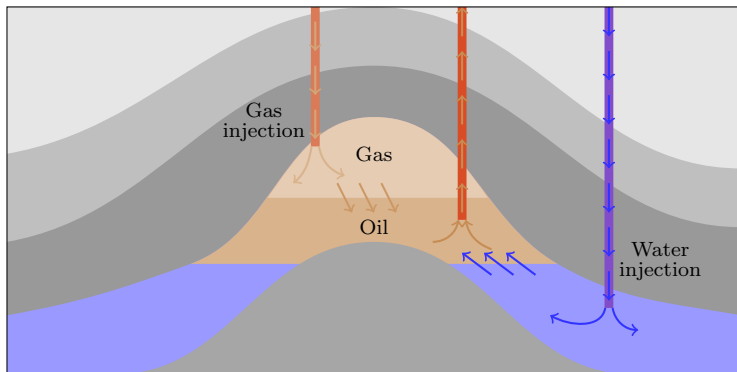
Production processes

Secondary production – maintaining reservoir flow

As pressure drops, less hydrocarbon is flowing. To maintain pressure and push more profitable hydrocarbons out, one starts injecting water or gas into the reservoir, possibly in an alternating fashion from the same well.

Production processes

Enhanced oil recovery

Even more crude oil can be extracted by gas injection (CO_2, natural gas, or nitrogen), chemical injection (foam, polymer, surfactants), microbial injection, or thermal recovery (cyclic steam, steam flooding, in-situ combustion), etc.

Why reservoir simulation?

To estimate reserves and support economic and operational decisions regarding recovery processes

To this end, reservoir engineers need to:
- ▶ understand reservoir and fluid behavior
- ▶ assess and optimize various recovery techniques
- ▶ quantify uncertainty
- ▶ test hypotheses and compare scenarios
- ▶ assimilate data

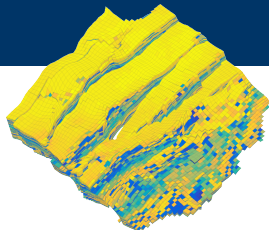Is a statutory requriement and recognized as evidence by banks and other funding organizations

Reservoir models

Somewhat simplified, consist of three parts:

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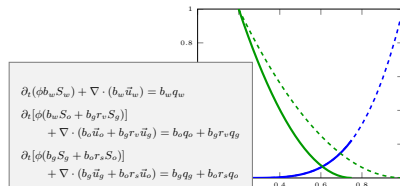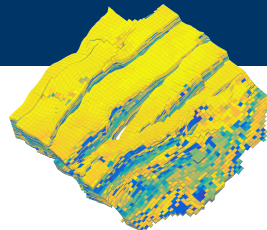1. a **geological model** – volumetric grid with cell/face properties describing the porous rock formation
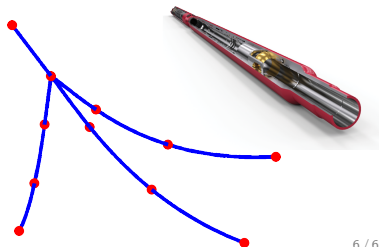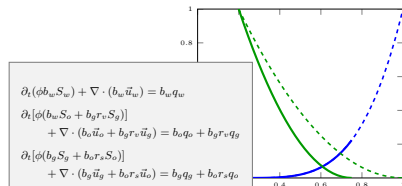
Reservoir models

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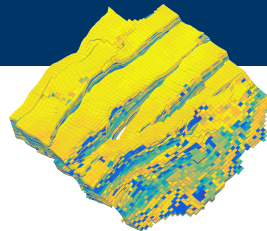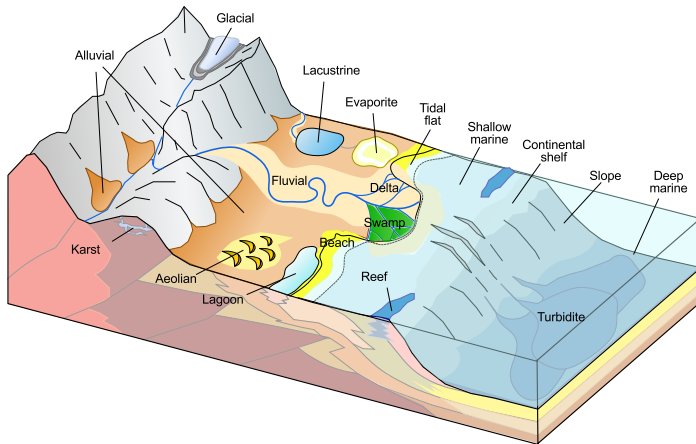
2. a **flow model** – describes how fluids flow in a porous medium (conservation laws + appropriate closure relations)

$$\partial_t(\phi b_w S_w) + \nabla \cdot (b_w \vec{u}_w) = b_w q_w$$

$$\partial_t[\phi(b_w S_o + b_g r_v S_g)]$$
$$+ \nabla \cdot (b_o \vec{u}_o + b_g r_v \vec{u}_g) = b_o q_o + b_g r_v q_g$$

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Reservoir models

Somewhat simplified, consist of three parts:

1. a **geological model** – volumetric grid with cell/face properties describing the porous rock formation

2. a **flow model** – describes how fluids flow in a porous medium (conservation laws + appropriate closure relations)

3. a **well model** – describes flow in and out of the reservoir, in the wellbore, flow control devices, surface facilities

$$\partial_t(\phi b_w S_w) + \nabla \cdot (b_w \vec{u}_w) = b_w q_w$$
$$\partial_t[\phi(b_o S_o + b_g r_v S_g)]$$
$$+ \nabla \cdot (b_o \vec{u}_o + b_g r_v \vec{u}_g) = b_o q_o + b_g r_v q_g$$
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$$+ \nabla \cdot (b_g \vec{u}_g + b_o r_s \vec{u}_o) = b_g q_g + b_o r_s q_o$$

Outline

Geologic model: sedimentary rocks

Mineral particles broken off by weathering and erosion

Transported by wind or water to a place where they settle and accumulate into a sediment, building up in lakes, rivers, sand deltas, lagoons, choral reefs, etc
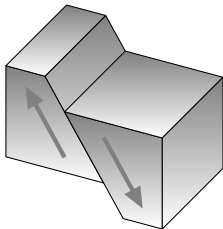
Geologic model: sedimentary rocks
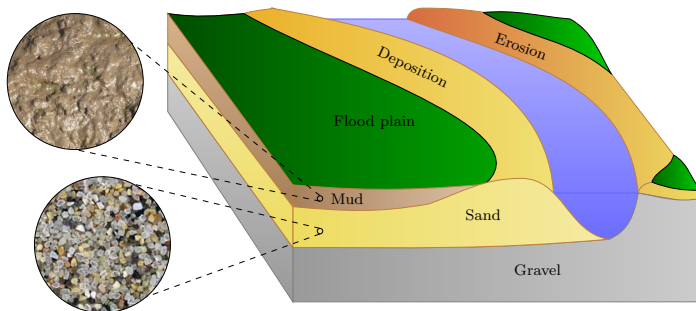
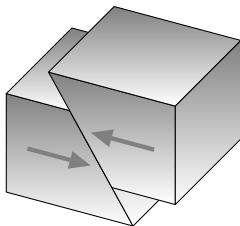Layered structure with different mixtures of rock types with varying grain size, mineral type, and clay content

Thin beds that stretch hundreds or thousands of meters, typically horizontally or at a small angle. Gradually buried deeper and consolidated

Geologic model: sedimentary rocks

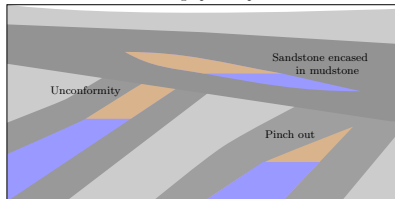Normal dip-slip fault   Reverse dip-slip fault   Strike-slip fault
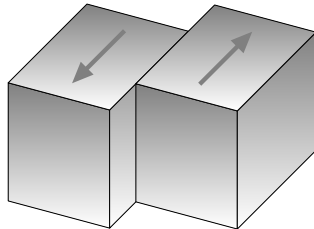
Geological activity will later fold, stretch, and fracture the consolidated rock
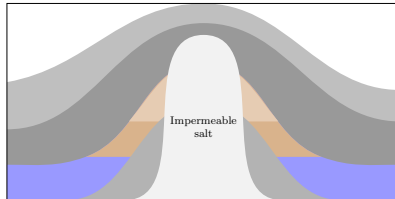
Structural trap: anticline

Gas

Oil

Impermeable rock

Permeable rock with brine
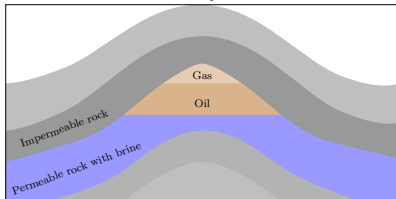
Stratigraphic traps

Sandstone encased in mudstone

Unconformity

Pinch out

Fault trap

Fault

Salt dome

Impermeable salt
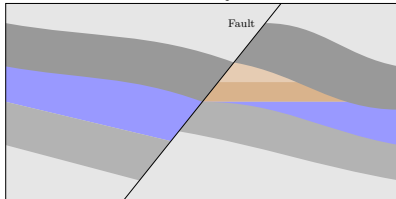
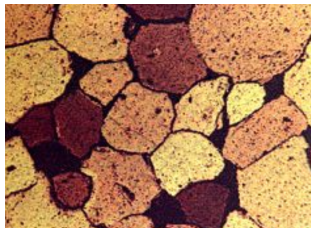Outcrops of sedimentary rocks from Svalbard, Norway. Length scale: ∼100 m

Layered geological structures typically occur on both large and small scales
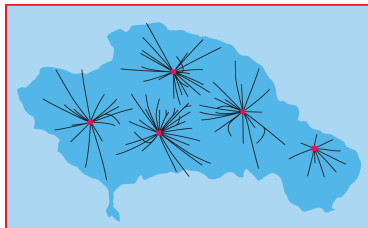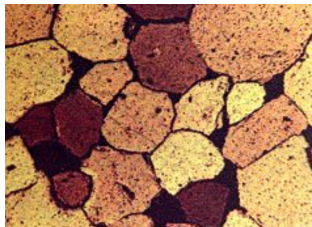
Porous media flow – a multiscale problem

The scales that impact fluid flow in subsurface rocks range from
- the micrometer scale of pores and pore channels
- via dm-m scale of well bores and laminae sediments
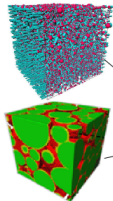- to sedimentary structures that stretch across entire reservoirs

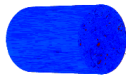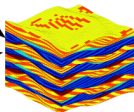Porous rocks are heterogeneous at all length scales (no scale separation)
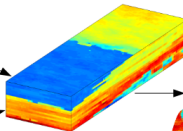

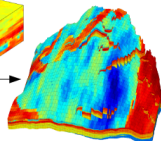$\longrightarrow$

Rock models [μm]

Core models [mm]

Geological models [cm]

Facies models [mm]
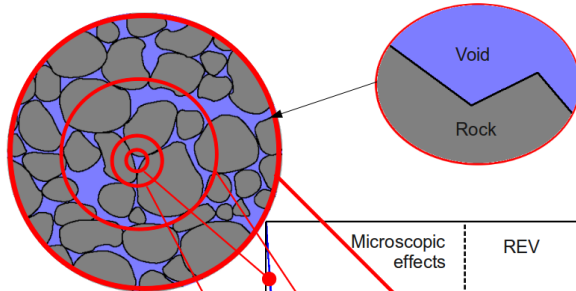
Simulation models [m]
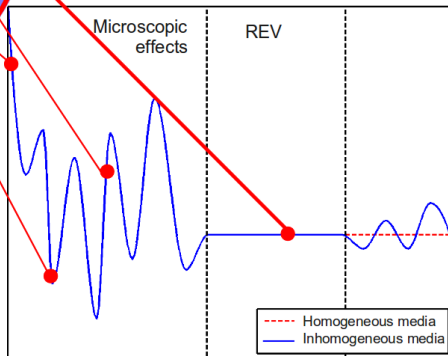
Microscopic

Mesoscopic

Macroscopic

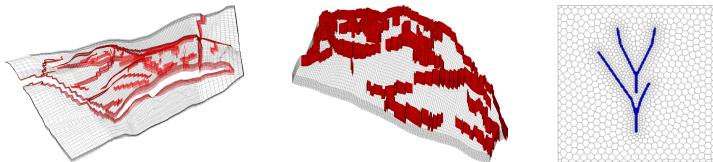# Flow model: representative elementary volume

Porosity:

$$\phi = \frac{V_v}{V_v + V_r}$$

The assumption of a representative elementary volume (REV) is essential in macro-scale modeling of porous media, here illustrated for porosity.
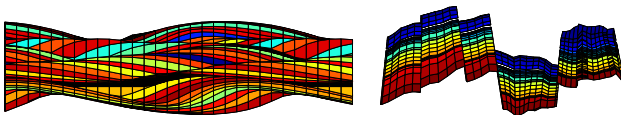
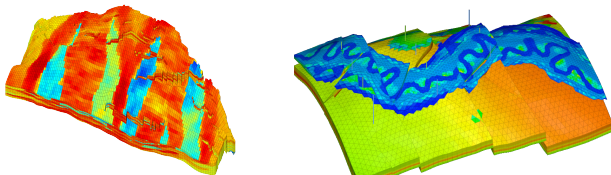# Grids: volumetric representation of the reservoir

The structure of the reservoir (geological surfaces, faults, etc) + well paths

The stratigraphy of the reservoir (sedimentary structures)

Petrophysical parameters (permeability, porosity, net-to-gross, . . .)

Grids: mimicking geological processes

Deposition

Erosion

Petrophysics

Deformation

Deposition

Erosion

Petrophysics

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Deposition

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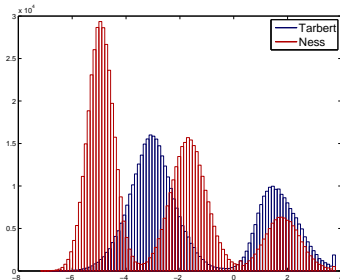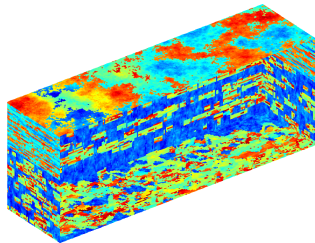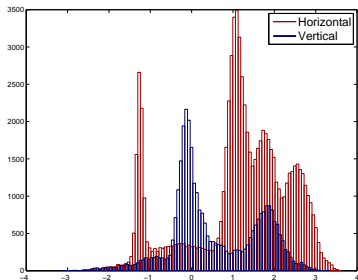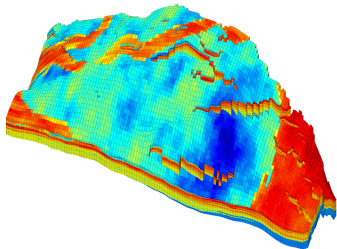
Research challenge: complex grid and strong heterogeneity
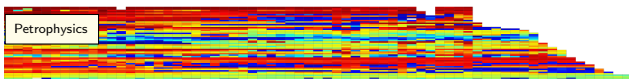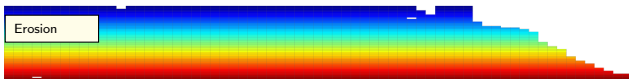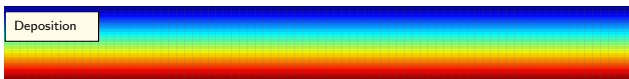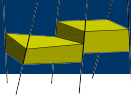
Complex, unstructured grids with many obscure challenges

- Grid dictated by geology, not chosen freely to maximize accuracy of numerical discretization
- Topology is generally unstructured, non-neighboring connections
- Cells deviate strongly from box shape, high aspect ratios, many faces/neighbors, small faces, . . .
- Potential inconsistencies: bilinear vs tetrahedral surfaces

Petrophysics:

- Many orders of magnitude variations
- Strong discontinuities
- No clear scale separation (long and short correlations)
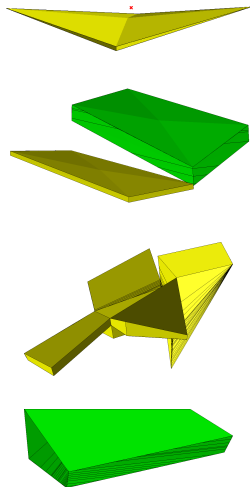
Outline

Governing equations for fluid flow

In its simplest form – two main principles

- Conservation of mass

$$\frac{\partial}{\partial t} \int_V m \, dx + \oint_{\partial V} \vec{F} \cdot \vec{n} \, ds = \int_V r \, dx$$

m=mass, $\vec{F}$=flow rate, r=fluid sources
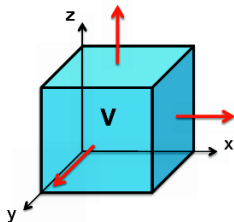
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- ▶ Darcy's law:

$$\vec{u} = -K(\nabla p - \rho g\nabla z)$$

empirical law for describing processes on an unresolved scale.
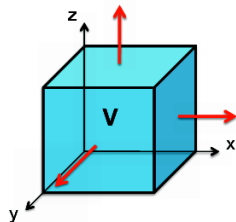
Similar to Fourier's law (heat) [1822], Ohm's law (electric current) [1827], Fick's law (concentration) [1855], except that we now have *two driving forces*

Darcy's law and permeability

In reservoir engineering:

$$\vec{u} = -\frac{\mathbf{K}}{\mu}\big(\nabla p - \rho g \nabla z\big)$$

Intrinsic permeability **K** measures ability to transmit fluids

Anisotropic and diagonal by nature, full tensor due to averaging.

Reported in units Darcy: $1\ \mathsf{d} = 9.869233 \cdot 10^{-13}\ \mathsf{m}^2$

Fluid velocity:

Darcy's law is formulated for *volumetric flux*, i.e., volume of fluid per total area per time. The *fluid velocity* is volume per area occupied by fluid per time, i.e., $\vec{v} = \frac{\vec{u}}{\phi}$.

Theoretical basis (M. K. Hubbert, 1956):

Darcy's law derived from the Navier–Stokes equations by averaging, neglecting intertial and viscous effects

Single-phase, incompressible flow

Model equations for single-phase flow:

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot \left(\rho\vec{u}\right) = q, \qquad \vec{u} = -\frac{\mathbf{K}}{\mu}\left(\nabla p - \rho g \nabla z\right)$$

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Assume constant density ρ, unit fluid viscosity μ, and neglect gravity g
$\longrightarrow$ flow equation on mixed form

$$\nabla \cdot \vec{u} = q, \qquad \vec{u} = -\mathbf{K}\nabla p$$

or as a Poisson equation with variable coefficients

$$-\nabla\big(\mathbf{K}\nabla p\big) = q$$

Single-phase, slightly compressible flow

Introduce compressibilities for rock and fluid

$$\frac{d\phi}{dp} = c_r \phi, \qquad \frac{d\rho}{dp} = c_f \rho$$

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Insert into conservation equation

$$\frac{\partial(\phi\rho)}{\partial t} = \nabla \cdot \left(\rho\frac{\mathbf{K}}{\mu}\nabla p\right)$$

$$\left[(c_r + c_f)\phi\rho\right]\frac{\partial p}{\partial t} = \frac{c_f\rho}{\mu}\,\nabla p \cdot \mathbf{K}\nabla p + \frac{\rho}{\mu}\,\nabla \cdot (\mathbf{K}\nabla p)$$

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If c_f is sufficiently small, so that $c_f\nabla p \cdot \mathbf{K}\nabla p \ll \nabla \cdot (\mathbf{K}\nabla p)$, we get

$$\frac{\partial p}{\partial t} = \frac{1}{\mu\phi c}\nabla \cdot (\mathbf{K}\nabla p), \qquad c = c_r + c_f$$

Numerical discretization

Assumption: a grid $\mathcal{G}$ consisting of a collection of polyhedral cells $\{\Omega_i\}$
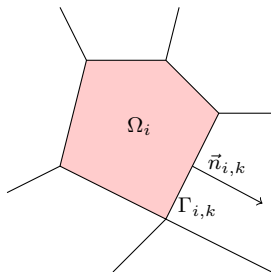
Numerical discretization

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Mass conservation per grid cell:

$$\int_{\Omega_i} \nabla \cdot \vec{u}\,dx = \oint_{\partial\Omega_i} \vec{u} \cdot \vec{n}\,ds = \int_{\Omega_i} q\,dx$$

$$\sum_k u_{i,k} = q_i$$

Pressure is cell-wise constant, flux is continuous across cell interfaces
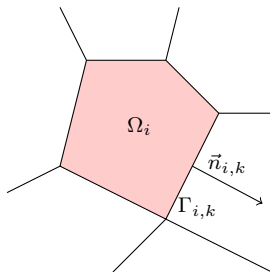
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Assume **K** is constant within each cell

$$u_{i,k} = -\int_{\Gamma_{ik}} \mathbf{K}\nabla p \cdot \vec{n}_{ik}\,ds$$
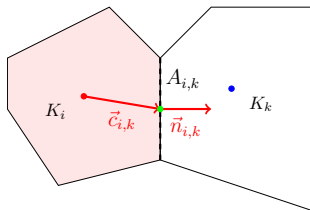
$$\approx A_k\,\mathbf{K}\,\frac{(p_i - \pi_{i,k})\vec{c}_{ik}}{|\vec{c}_{ik}|^2} \cdot \vec{n}_{ik}$$

$$= T_{i,k}(p_i - \pi_{i,k})$$

Numerical discretization

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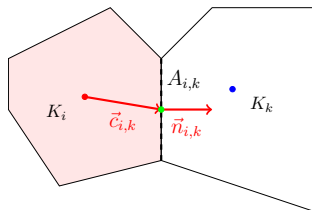
$$= T_{i,k}\big(p_i - \pi_{i,k}\big)$$

Next, we use continuity of flux and pressure to eliminate the interface pressures

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Numerical discretization

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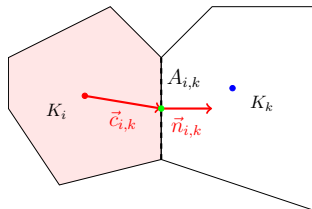
Next, we use continuity of flux and pressure to eliminate the interface pressures

$$u_{i,k} = T_{ik}\big(p_i - p_k\big)$$

Mass conservation $q_i = \sum_k u_{i,k}$ gives a linear system

$$\mathbf{A}\boldsymbol{p} = \boldsymbol{q}, \qquad \text{where } A_{ij} = \begin{cases} \sum_j T_{ij}, & k = i, \\ -T_{ik}, & k \neq i. \end{cases}$$

Research challenge: efficient solvers
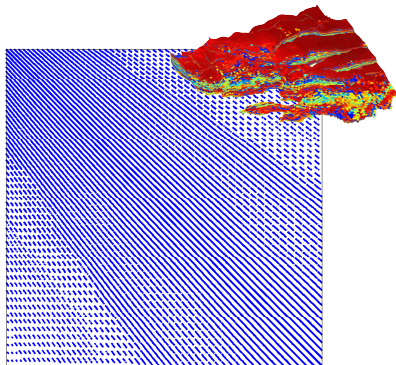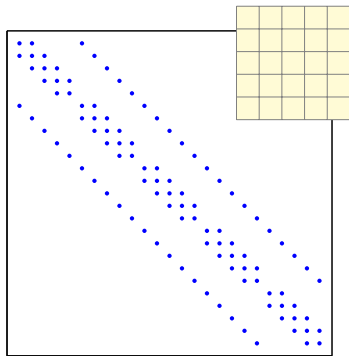
Large coefficient variations, complex sparsity patterns, etc. Call for efficient iterative solvers and preconditioning methods $\longrightarrow$ good test problems for multigrid methods

Research challenge: consistent discretizations

Problem: standard finite-volume methods are not consistent unless the grid is **K** orthogonal
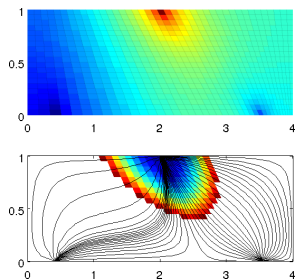
Research challenge: consistent discretizations

Problem: standard finite-volume methods are not consistent unless the grid is **K** orthogonal

$$u_{ik} = -\int_{\Gamma_{ik}} \left(K_{xx}\partial_x p + K_{xy}\partial_y p + K_{xz}\partial_z p \right) ds$$

Here, $\partial_y p$ and $\partial_z p$ cannot be estimated from p_i and p_k $\longrightarrow$ transverse flux $K_{xy}p_y$ and $K_{xz}p_z$ neglected $\longrightarrow$ inconsistent scheme

Research challenge: consistent discretizations

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Many methods developed to amend this

- (mortar) mixed finite elements
- multipoint flux approximation (MFPA)
- mimetic finite difference
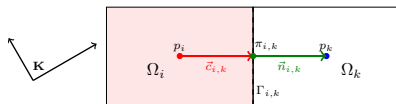- vertex approximate gradient (VAG)
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Research challenge: consistent discretizations

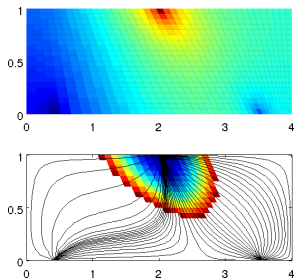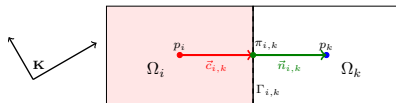Problem: standard finite-volume methods are not consistent unless the grid is **K** orthogonal
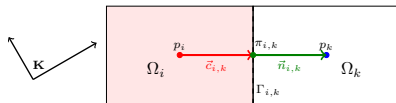
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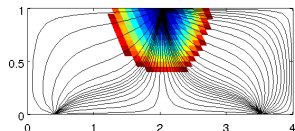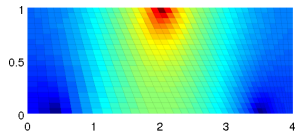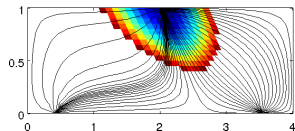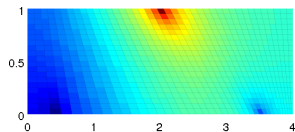
Example: comparison of consistent methods

Example: 3D Voronoi grid adapting to branching well. Anisotropic and spatially varying permeability
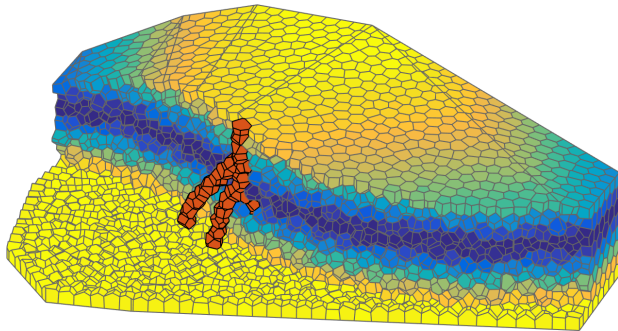
| Method	dof	nnz	ratio	cond
TPFA	9026	126002	13.96	9.64e+02
NTPFA	11920	280703	23.55	2.92e+07
MFD	60321	1538305	25.50	9.37e+15
VEM1	52350	3404977	65.04	4.91e+11
VEM2	227459	38770593	170.45	—

What can you do with single-phase flow?

Run basic diagnostics of your model to establish basic timelines, volumetric connections, measures of dynamic heterogeneity, etc
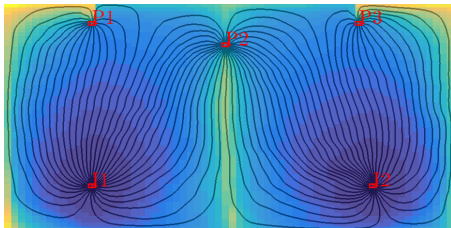
Forward time of flight
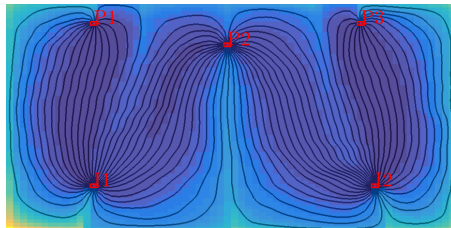
Residence time

Can be computed by tracing streamline or by finite-volume methods solving steady transport equations $\vec{u} \cdot \nabla h = f(x)$

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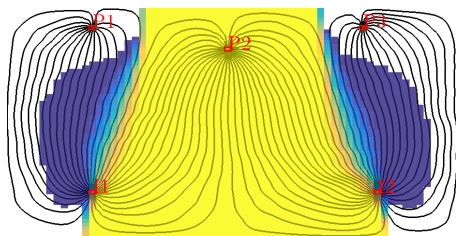
Influence regions
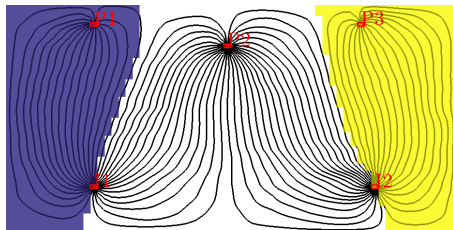
Well-pair regions

Can be computed by tracing streamline or by finite-volume methods solving steady transport equations $\vec{u} \cdot \nabla h = f(x)$

Flow diagnostics
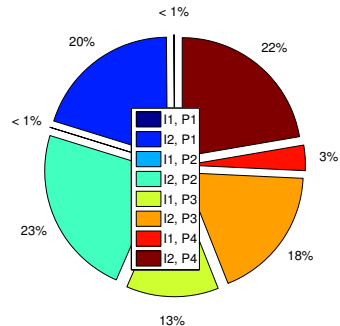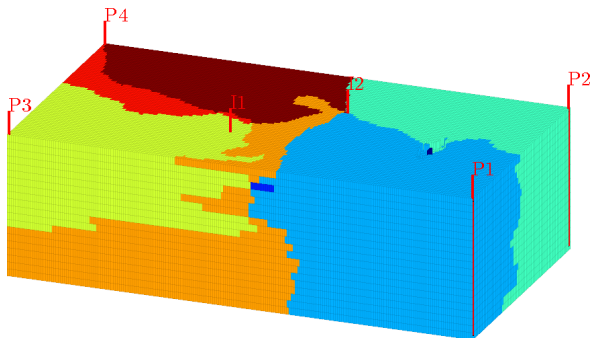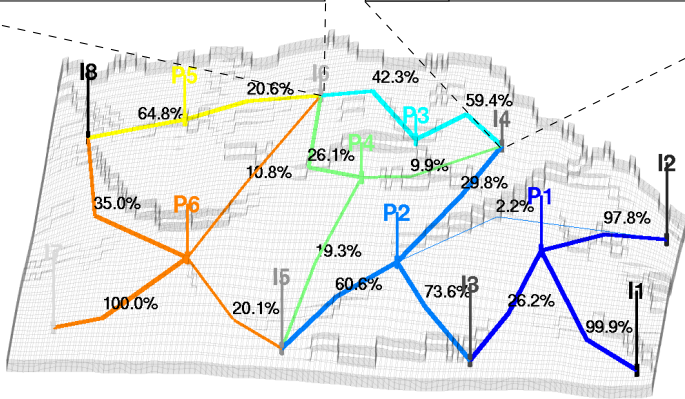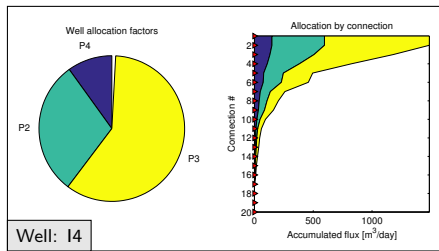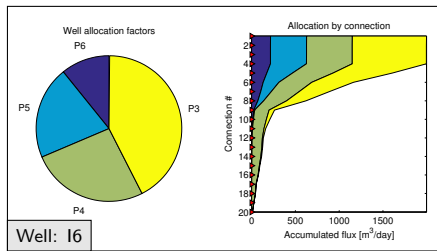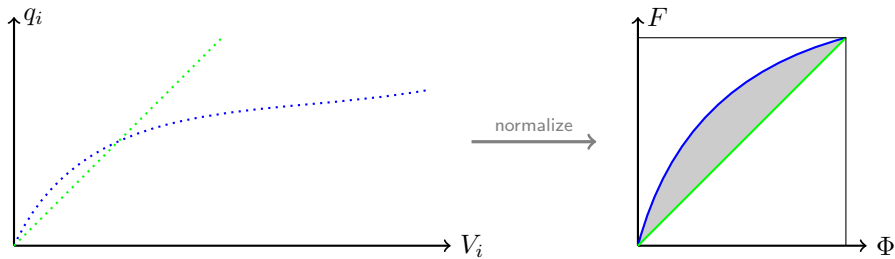
Flow diagnostics

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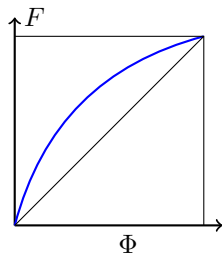
F-Φ diagram
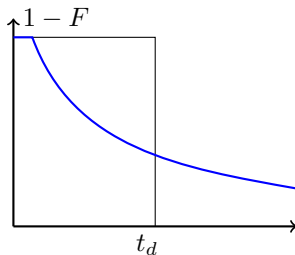
Fractional recovery
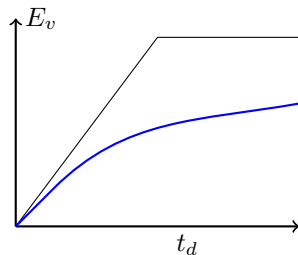
Sweep efficiency

Model reduction: flow-based upscaling

$$-\nabla \cdot (\mathbf{K}\nabla p) = f, \qquad \text{in } \Omega$$

Subdivide grid into coarse blocks. For each block B, we seek a tensor $\mathbf{K}^*$ such that

$$\int_B \mathbf{K}\nabla p \, dx = \mathbf{K}^* \int_B \nabla p \, dx,$$

That is, we use Darcy's law on the coarse scale

$$\bar{u} = -\mathbf{K}^*\overline{\nabla p}$$

to relate the net flow rate $\bar{u}$ through B to the average pressure gradient $\overline{\nabla p}$ inside B.
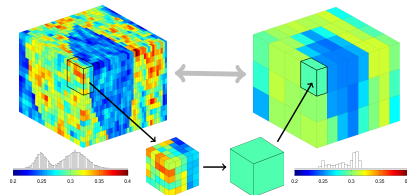
Typical approach: flow-based upscaling
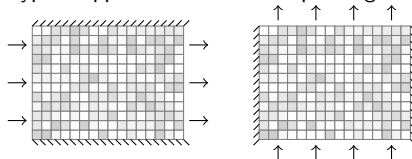
Many alternatives, few are sufficiently accurate and robust

More about this on Wednesday

Outline

Multiphase flow

Hydrocarbon typically consists of different chemical species like methane, ethane, propane, etc. Common modelling practice to group fluid components into **phases**, i.e., a mixture of components having similar flow properties.

Most common phases:

aqueous, liquid, and vapor
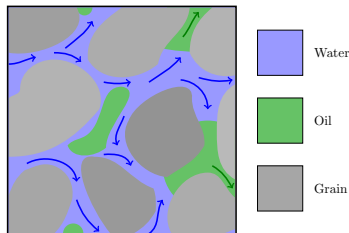
Fundamental physics: wettability

Immiscible phases separated by a infinitely thin surface having associated surface tension
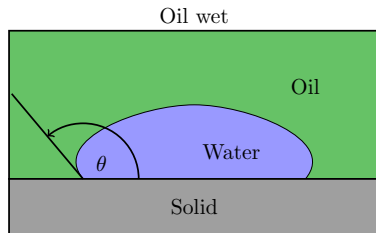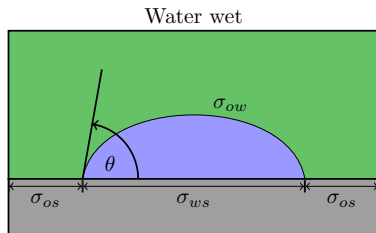
Contact angle θ: determined by balance of adhesive and cohesive forces

Young's equation (energy balance): $\sigma_{ow} \cos \theta = \sigma_{os} - \sigma_{ws}$

Water generally shows greater affinity than oil to stick to the rock surface $\longrightarrow$ reservoirs are predominantly water-wet systems

Different equilibrium pressure in two phases separated by curved interface:

$$p_c = p_n - p_w = \underbrace{\frac{2\pi r\, \sigma \cos\theta}{\pi r^2}}_{\text{upward force}} = \frac{2\sigma \cos\theta}{r} = \underbrace{\frac{\pi r^2\, gh(\rho_l - \rho_a)}{\pi r^2}}_{\text{downward force}} = \Delta\rho g h$$

Fundamental physics: drainage (primary migration)

Saturation: fraction of pore volume filled by a given fluid phase

Drainage: non-wetting fluid displacing wetting fluid, controlled by widest non-invaded pore throat

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Drainage: non-wetting fluid displacing wetting fluid, controlled by widest non-invaded pore throat

Fundamental physics: imbibition (hydrocarbon recovery)

Imbibition: wetting fluid displaces non-wetting fluid, controlled by the size of the *narrowest* non-invaded pore.

Will not follow the same capillary curve $\longrightarrow$ **hysteresis** (cause: trapped oil droplets, different wetting angle for advancing and receding interfaces)
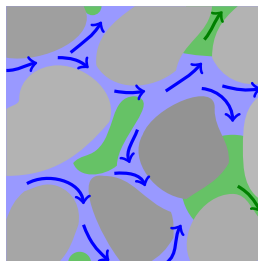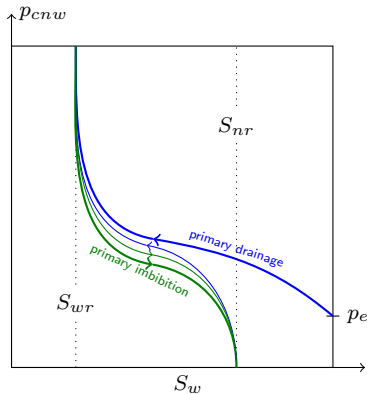
EOR: inject substances to alter wetting properties to mobilize immobile oil, $S_{or} \rightarrow 1$

Extensions of model equations to multiphase flow
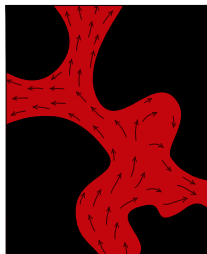
Three-phase Darcy velocities (Muskat, 1936):

$$\vec{u}_\alpha = -\frac{\mathbf{K}_\alpha(S_\alpha)}{\mu_\alpha}\big(\nabla p_\alpha - \rho_\alpha g \nabla z\big)$$

Assuming each phase consists of only one component, the mass-balance equations for each phase read (Muskat, 1945):

$$\frac{\partial(\phi \rho_\alpha S_\alpha)}{\partial t} + \nabla \cdot \big(\rho_\alpha \vec{u}_\alpha\big) = q_\alpha$$
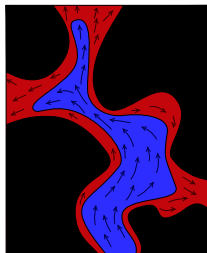
Macro-scale capillarity concept (Leverett, 1941):

$$p_c(S_w) = J\sqrt{\frac{\phi}{K}}\,\sigma \cos\theta$$

Relative permeability

Effective permeability experienced by one phase is reduced by the presence of other phases. Relative permeabilities

$$k_{r\alpha} = k_{r\alpha}(S_{\alpha_1}, \ldots, S_{\alpha_m}),$$

are nonlinear functions that attempt to account for this effect. Notice that

$$\sum_{\alpha} k_{r\alpha} < 1$$
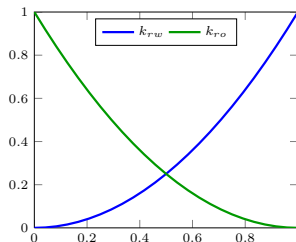
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This gives Darcy's law on the form
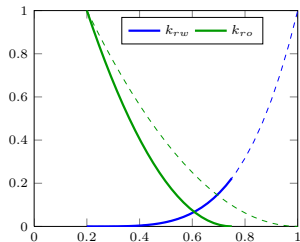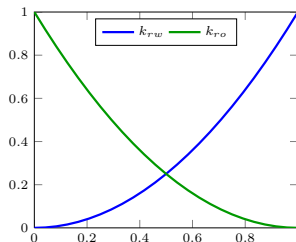
$$\vec{u}_\alpha = -\frac{\mathbf{K} k_{r\alpha}}{\mu_\alpha}\left(\nabla p_\alpha - \rho_\alpha g \nabla z\right)$$
$$= -\mathbf{K}\lambda_\alpha\left(\nabla p_\alpha - \rho_\alpha g \nabla z\right)$$

General flow equations for two-phase flow

Gathering the equations, we have

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t} + \nabla \cdot \left(\rho_\alpha \vec{u}_\alpha\right) = q_\alpha, \qquad \alpha = \{w, n\}$$

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$$p_c = p_n - p_w, \qquad S_w + S_n = 1$$

Commercial reservoir simulators: insert functional relationships $p_c = P_c(S_w)$ and ρ_α and ϕ as function of p_α, and discretize with backward Euler in time and the two-point scheme in space

In academia: common practice to rewrite the equations to better reveal their mathematical nature

Fractional flow formulation

Choose S_w and p_n as primary unknowns, consider incompressible flow (i.e., ρ is constant and can be divided out)

$$\phi \frac{\partial S_\alpha}{\partial t} + \nabla \cdot \vec{u}_\alpha = q_\alpha.$$

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Sum mass-conservation equations:

$$\phi \frac{\partial}{\partial t}(S_n + S_w) + \nabla \cdot (\vec{u}_n + \vec{u}_w) = q_n + q_w$$

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Sum Darcy equations

$$\vec{u} = \vec{u}_n + \vec{u}_w = -(\boldsymbol{\lambda}_n + \boldsymbol{\lambda}_w)\nabla p_n + \boldsymbol{\lambda}_w \nabla p_c + (\boldsymbol{\lambda}_n \rho_n + \boldsymbol{\lambda}_w \rho_w)g\nabla z$$

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Inserted into $\nabla \cdot \vec{u} = q$ gives pressure equation

$$-\nabla \cdot (\lambda \mathbf{K}\nabla p_n) = q - \nabla\big[\boldsymbol{\lambda}_w \nabla p_c + (\boldsymbol{\lambda}_n \rho_n + \boldsymbol{\lambda}_w \rho_w)g\nabla z\big]$$

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Inserted into $\nabla \cdot \vec{u} = q$ gives pressure equation

$$\underbrace{-\nabla \cdot (\lambda\mathbf{K}\nabla p_n)}_{\text{Poisson}} = q - \underbrace{\nabla\big[\boldsymbol{\lambda}_w\nabla p_c + (\boldsymbol{\lambda}_n\rho_n + \boldsymbol{\lambda}_w\rho_w)g\nabla z\big]}_{\text{only function of } S_w}$$

Fractional flow formulation

Multiply phase velocity by mobility of other phase and subtract

$$\lambda_n \vec{u}_w - \lambda_w \vec{u}_n = \lambda_w \lambda_n \mathbf{K} \left[\nabla p_c + (\rho_w - \rho_n) g \nabla z \right]$$

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Solve for $\vec{u}_w$ and insert into conservation equation

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \left[f_w \left(\vec{u} + \boldsymbol{\lambda}_n \Delta \rho g \nabla z \right) \right] = q_w - \nabla \cdot \left(f_w \boldsymbol{\lambda}_n P_c' \nabla S_w \right)$$

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Setting $P_c \equiv 0$ and $g \equiv 0$ for simplicity

$$-\nabla \big(\mathbf{K} \lambda(S) \nabla p \big) = q, \qquad \vec{u} = -\mathbf{K} \lambda(S) \nabla p,$$
$$\phi \partial_t S + \nabla \cdot (\vec{u} f(S)) = 0$$

System of one **elliptic** pressure equation and one **hyperbolic** saturation equation. Typically: solved sequentially with specialized methods

Buckley–Leverett solution for 1D displacement
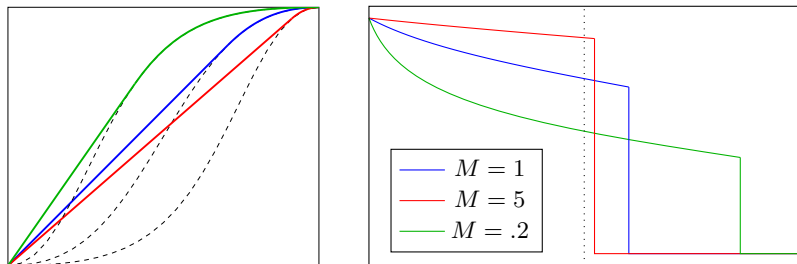
$$S_t + f(S)_x = q, \qquad f(S) = \frac{S^2}{S^2 + M(1-S)^2}, \quad M = \mu_w/\mu_n$$

Here, $M = .2$ gives poor local displacement efficiency, $M = 5$ gives very good

Simulation examples: quarter-five spot
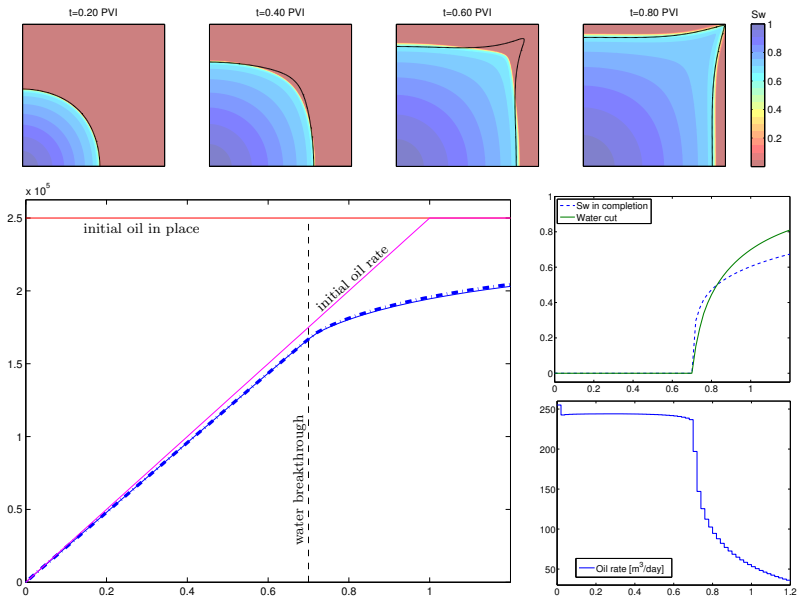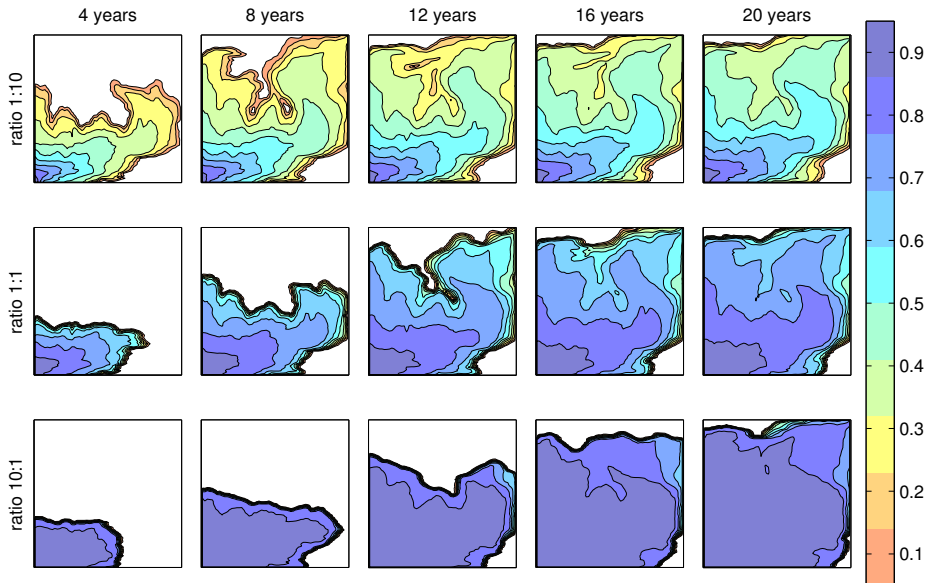
Simulation examples: quarter-five spot

Simulation examples: quarter-five spot

150 days

670 days

810 days

1020 days

1500 days

4500 days

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
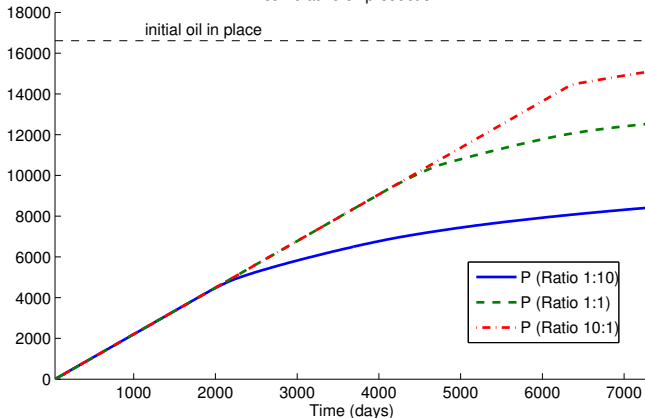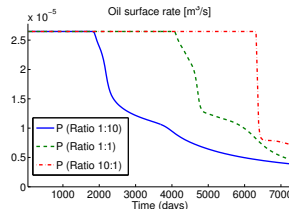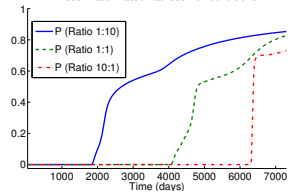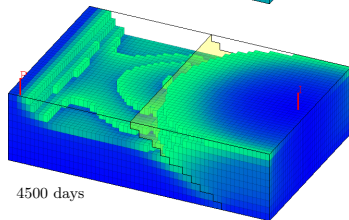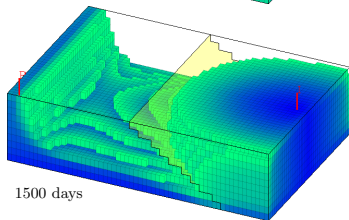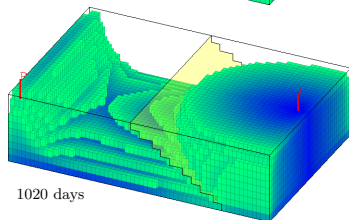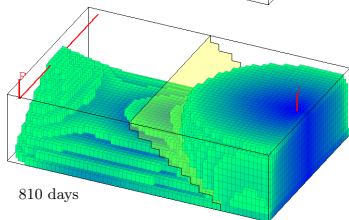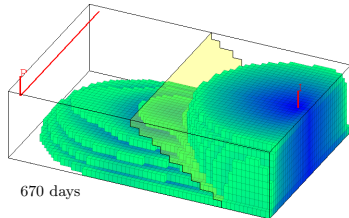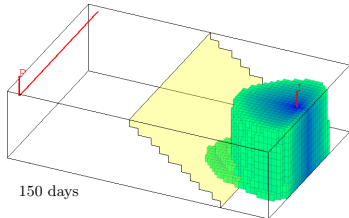
High permeability on top — Low permeability on top
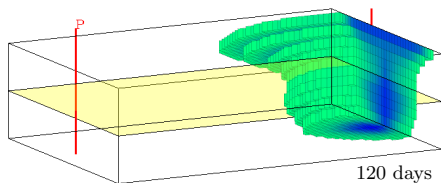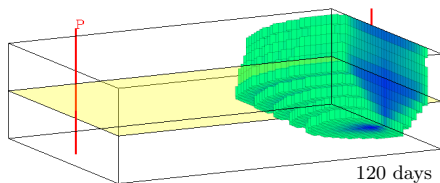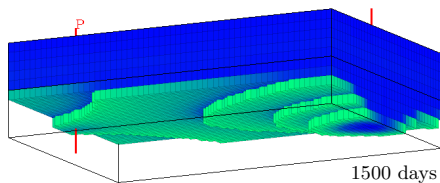
120 days — 120 days
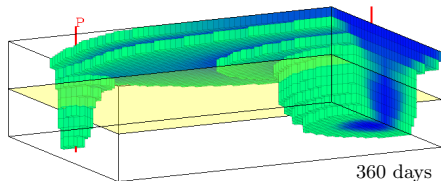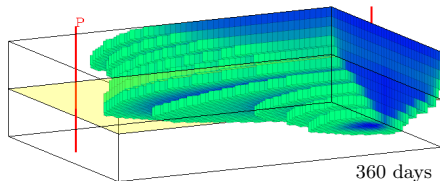
360 days — 360 days

1500 days — 1500 days

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Numerical errors

There are two main sources of errors one should be wary of

- Operator splitting errors introduced by solving flow and transport separately

- Grid-orientation errors introduced by numerical method

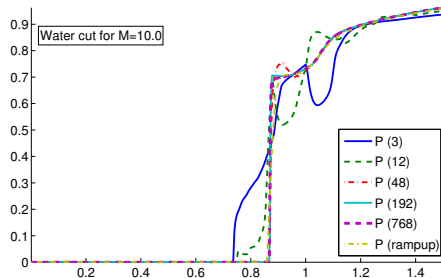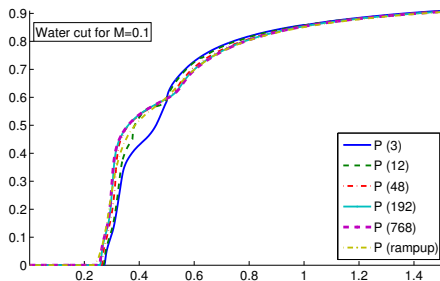There are two main sources of errors one should be wary of
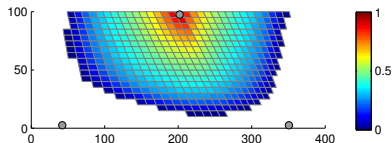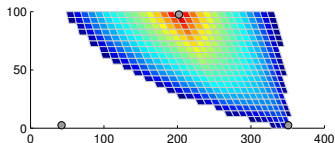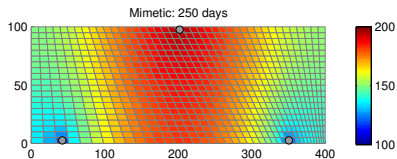
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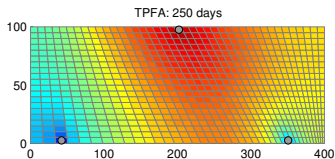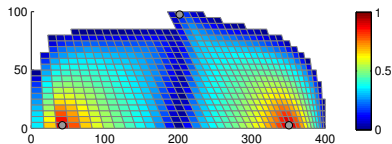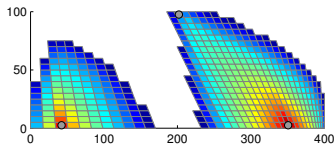There are two main sources of errors one should be wary of

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Numerical errors

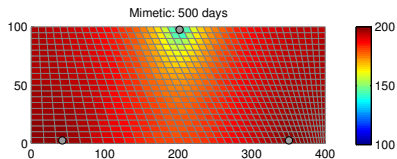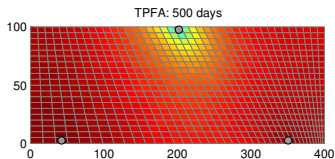There are two main sources of errors one should be wary of

- Operator splitting errors introduced by solving flow and transport separately

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Outline

System with N phases and M components

Notation: c_α^ℓ mass fraction of component ℓ in phase α

Equations: conservation for phases or components?
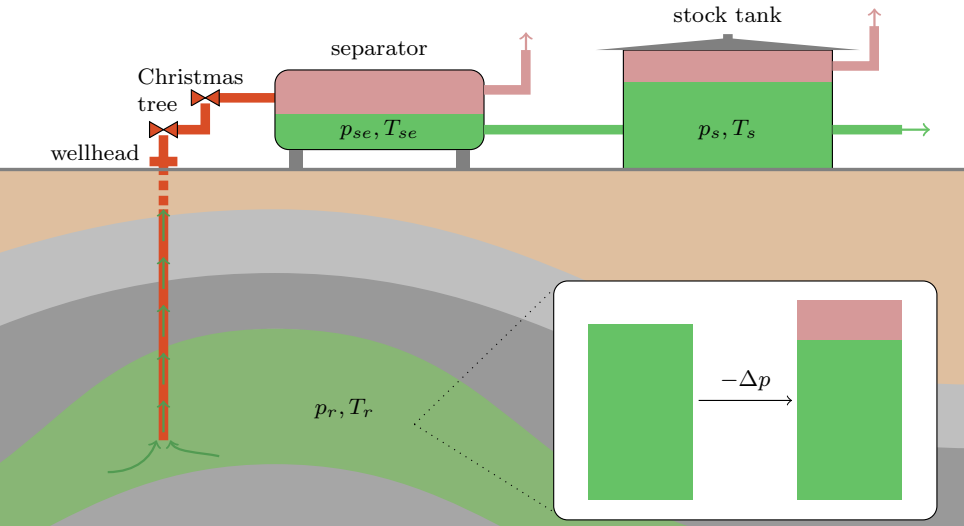
Choose components to avoid source terms for mass transfer

$$\frac{\partial}{\partial t}\left(\phi \sum_\alpha c_\alpha^\ell \rho_\alpha S_\alpha\right) + \nabla \cdot \left(\sum_\alpha c_\alpha^\ell \rho_\alpha \vec{u}_\alpha + \vec{J}_\alpha^\ell\right) = \sum_\alpha c_\alpha^\ell \rho_\alpha q_\alpha,$$

where J is diffusion, e.g., Fickian

$$\vec{J}_\alpha^\ell = -\rho_\alpha S_\alpha \mathbf{D}_\alpha^\ell \nabla c_\alpha^\ell,$$

PVT behavior of petroleum fluids

Phase diagrams for a single component

Gibbs' phase rule (without reactions):

$$F = 2 + n_c - n_p$$

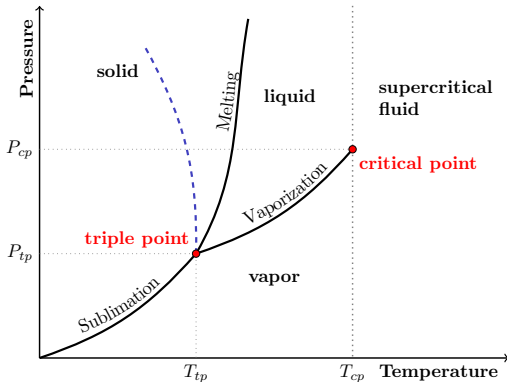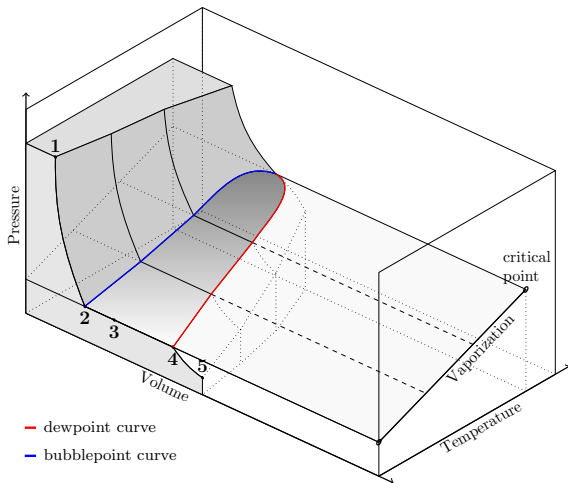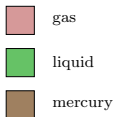F = degrees of freedom, n_c = components, and n_p = phases

Phase diagrams for a single component

1: all liquid

4: last droplet

2: first bubble

5: all gas

3: gas and liquid

gas

liquid

mercury

— dewpoint curve
— bubblepoint curve

Pressure

Volume

Temperature

Vaporization

critical point
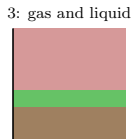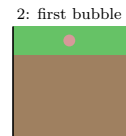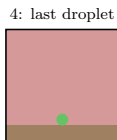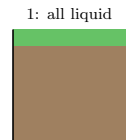
Binary substances

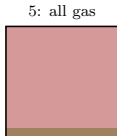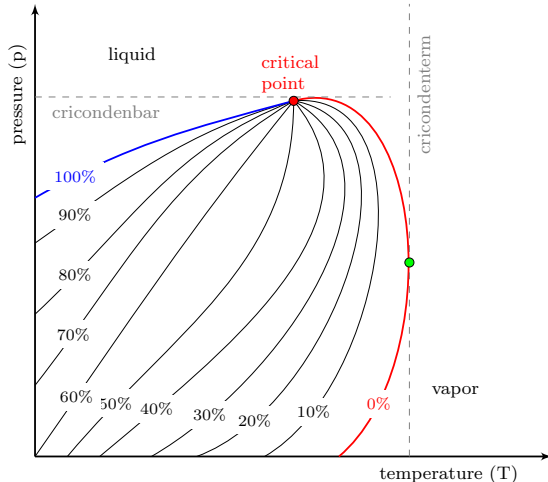Bubblepoint and vaporization curve no longer coincide. Phase diagram depends on the composition of the binary mixture
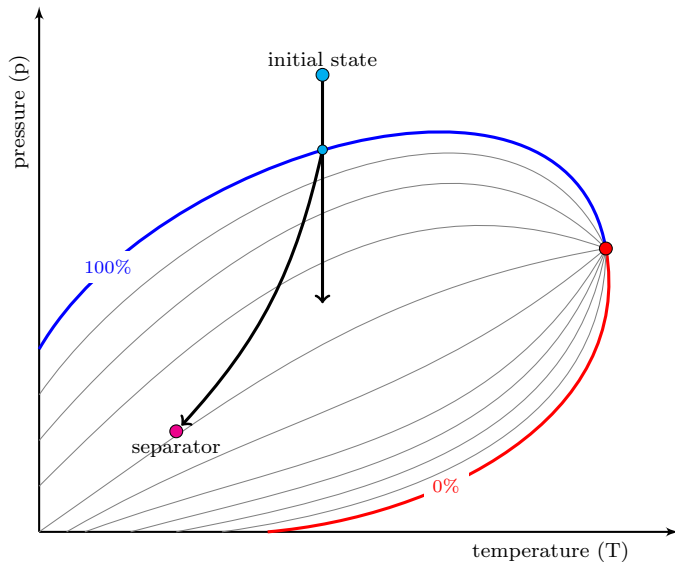
Oil reservoirs: volatile oil
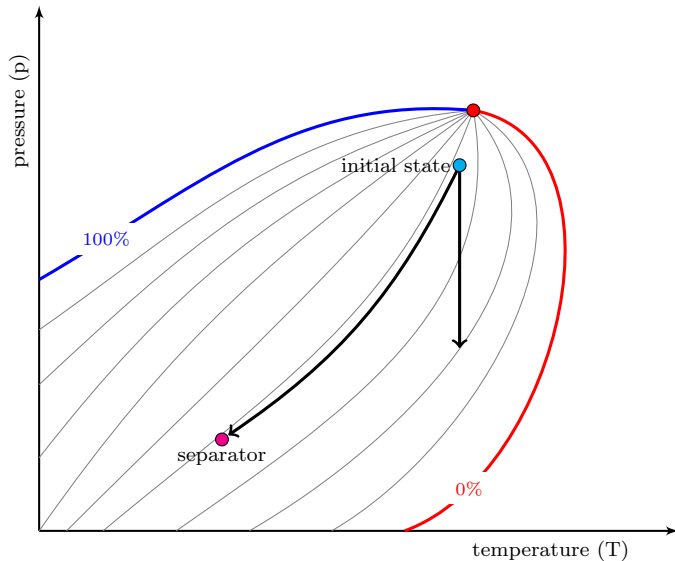
Black-oil models

Three phases: aqueous, liquid, and vapor

Hydrocarbon components lumped together to a light 'gas' and a heavier 'oil' pseudocomponent at surface conditions

	W	O	G
A	X		
L		X	X
V		X	X

Simple PVT:

$$\rho_W = \frac{\rho_{ws}}{B_w} = b_w \rho_{ws}$$

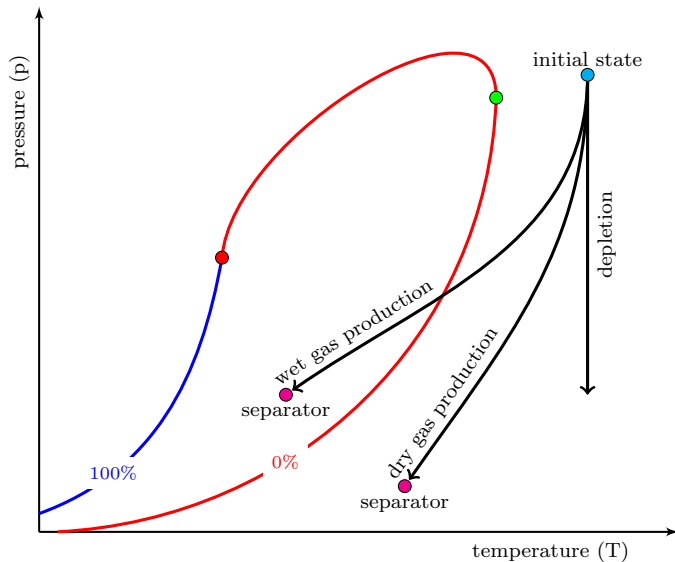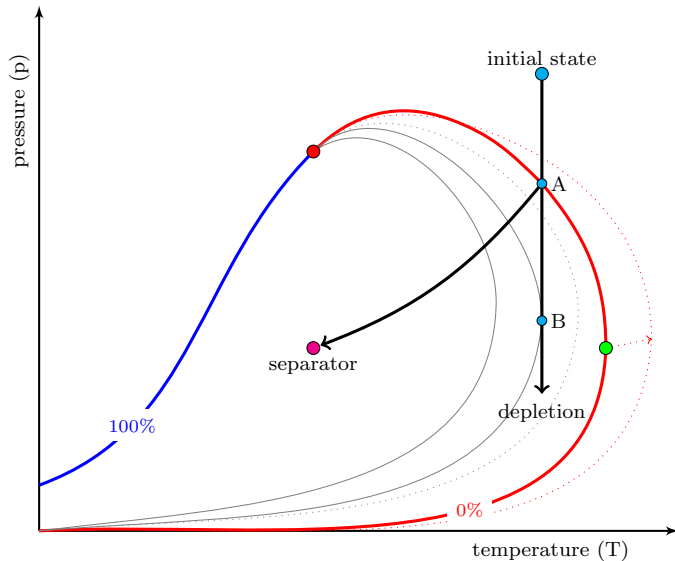$$\rho_O = \frac{\rho_{os} + R_s \rho_{gs}}{B_o} = b_o(\rho_{os} + R_s \rho_{gs})$$

$$\rho_G = \frac{\rho_{gs} + R_v \rho_{os}}{B_g} = b_g(\rho_{gs} + R_v \rho_{os})$$

Formation-volume factor: $B_\alpha = \frac{V_\alpha}{V_\alpha^s} = \frac{\rho_\alpha^s}{\rho_\alpha}$
Shrinkage/expansion factor: $b_\alpha = 1/B_\alpha$
Dissolved gas in oil: R_s
Oil vaporized in gas: R_v
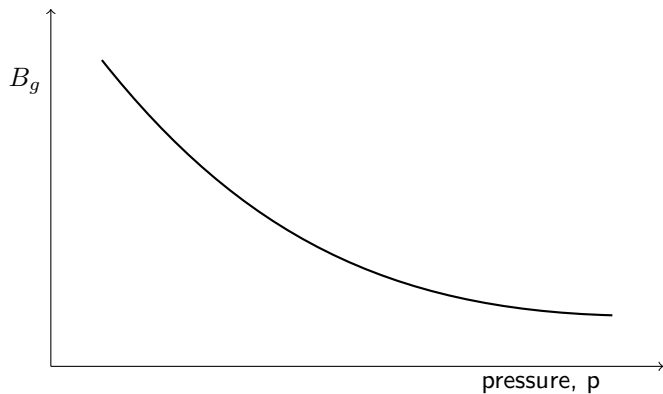
reservoir

surface
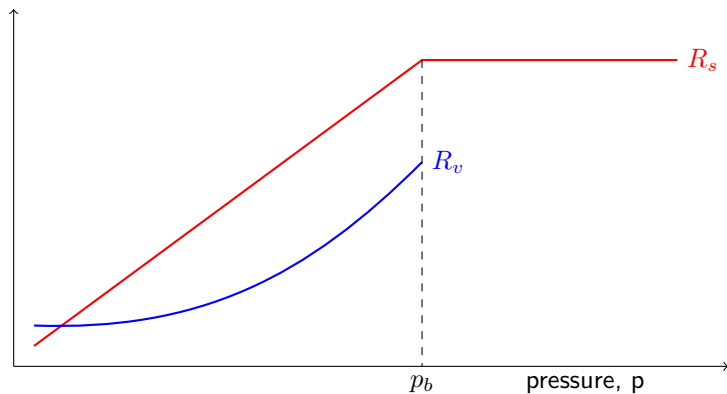
Black-oil: PVT functions
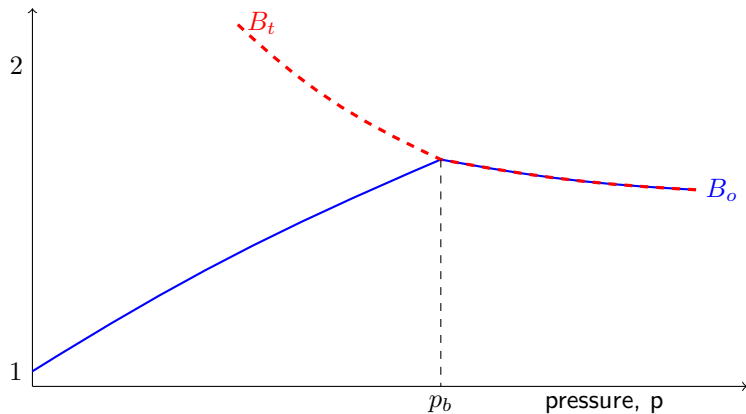
Black-oil: PVT functions
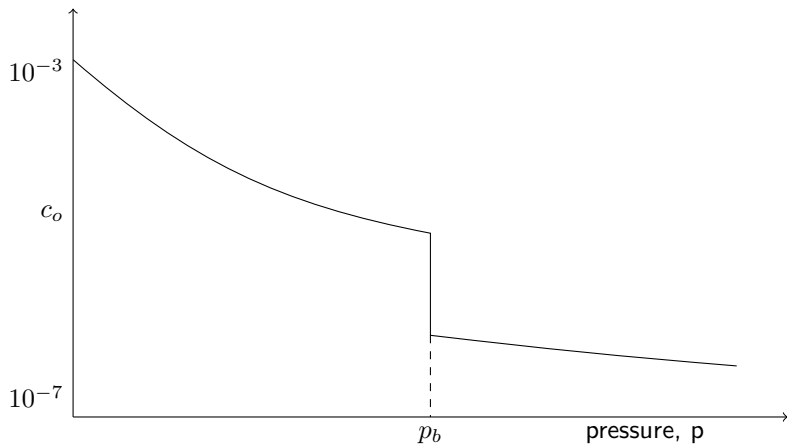
Black-oil: PVT functions
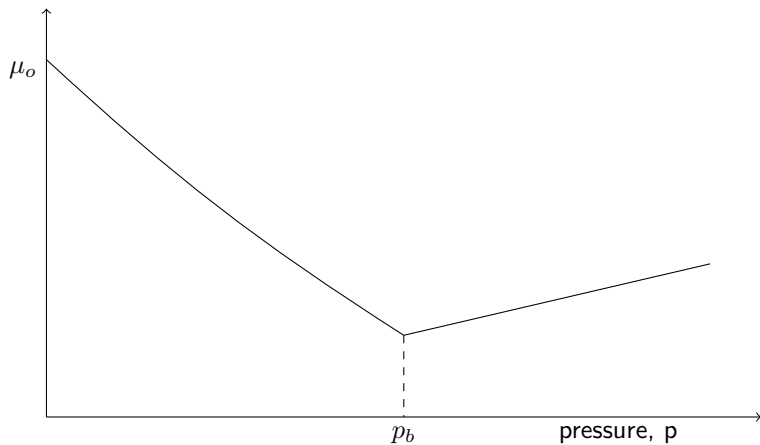
The black-oil equations

Conservation equations:

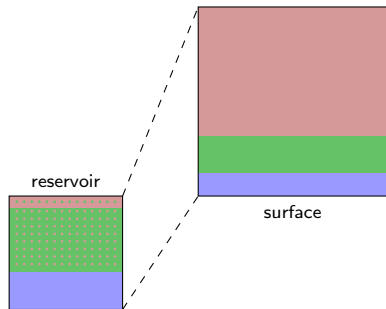$$\partial_t\big(\phi b_o S_o\big) + \nabla \cdot \big(b_o \vec{u}_o\big) = b_o q_o$$
$$\partial_t\big(\phi b_w S_w\big) + \nabla \cdot \big(b_w \vec{u}_w\big) = b_w q_w$$
$$\partial_t\big[\phi\big(b_g S_g + b_o R_s S_o\big)\big] + \nabla \cdot \big(b_g \vec{u}_g + b_o R_s \vec{u}_o\big) = b_g q_g + b_o R_s q_o$$

Darcy's law:

$$\vec{u}_\alpha = -\frac{\mathbf{K}_\alpha(S_\alpha)}{\mu_\alpha}\big(\nabla p_\alpha - \rho_\alpha g \nabla z\big)$$

Closure relationships

$$S_o + S_w + S_g = 1$$

Plus expressions for capillary pressures and relative permeabilities

Three-phase relative permeability

In principle, one *could* imagine all three phases flowing simultaneously. However, more common to assume that flow is essentially two-phase
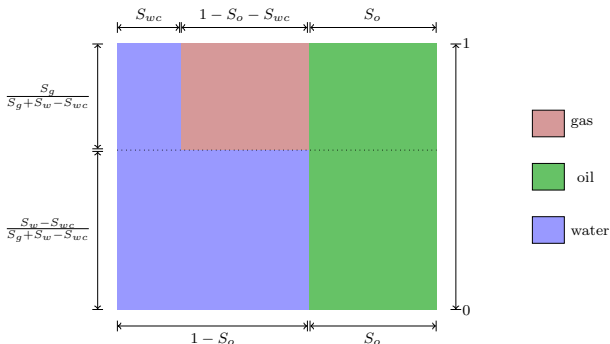
Relative permeability of oil:

$$k_{ro}(S_g, S_w) = \frac{S_g \; k_{rog}(S_o)}{S_g + S_w - S_{wc}} + \frac{(S_w - S_{wc}) \; k_{row}(S_o)}{S_g + S_w - S_{wc}},$$

Relative permeability for oil
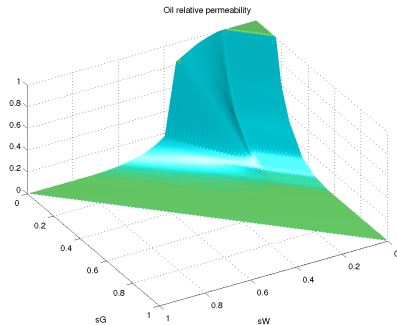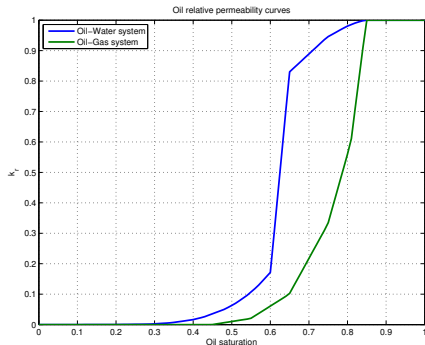
J. E. Killough (1995). Ninth SPE comparative solution project: A reexamination of black-oil simulation. SPE Reservoir Simulation Symposium

Formation-volume factors (inverse densities):
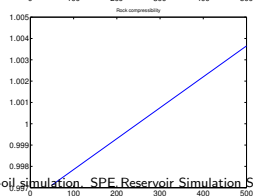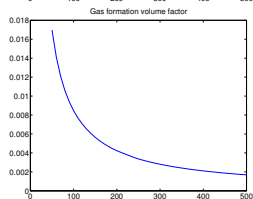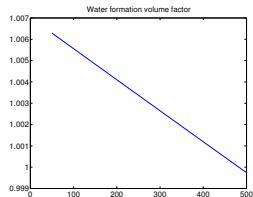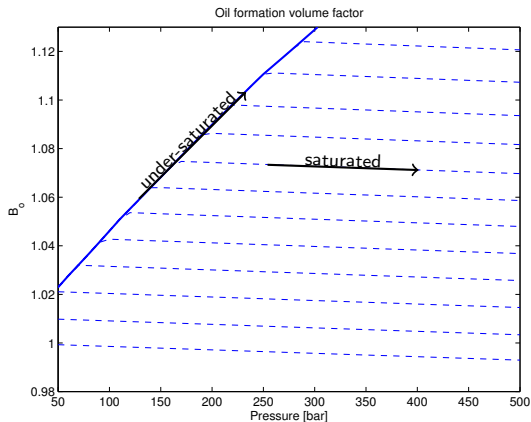
J. E. Killough (1995). Ninth SPE comparative solution project: A reexamination of black-oil simulation. SPE Reservoir Simulation Symposium

Example: fluid model from SPE9

Viscosities:
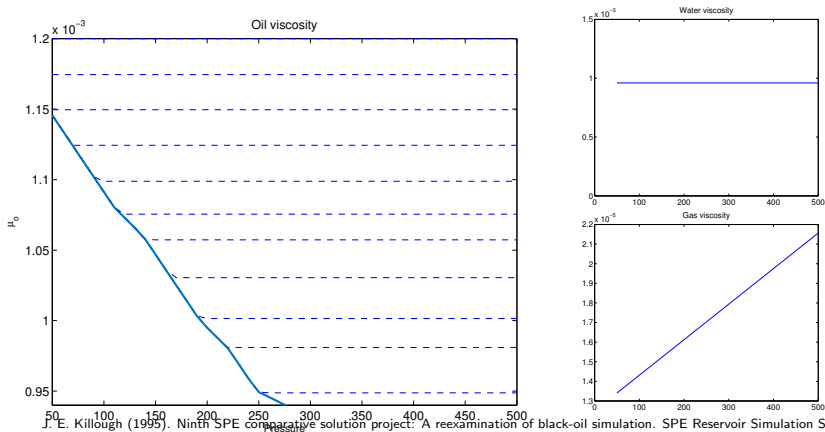
J. E. Killough (1995). Ninth SPE comparative solution project: A reexamination of black-oil simulation. SPE Reservoir Simulation Symposium

Outline

Wells: flow in and out of the reservoir
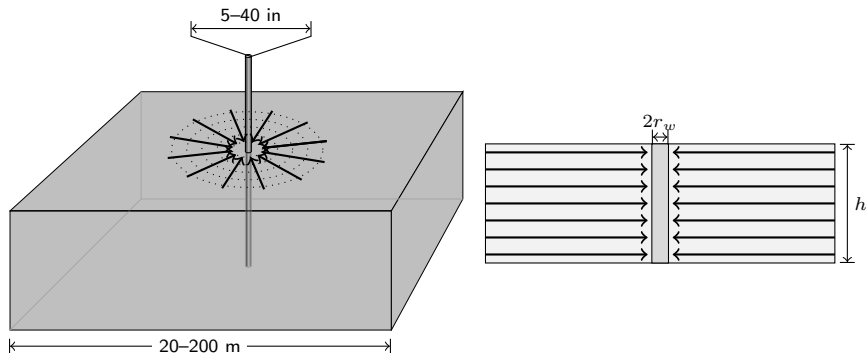
Inflow and outflow take place on a *subgrid* scale, with large variations in pressure over short distances.

Wells: flow in and out of the reservoir
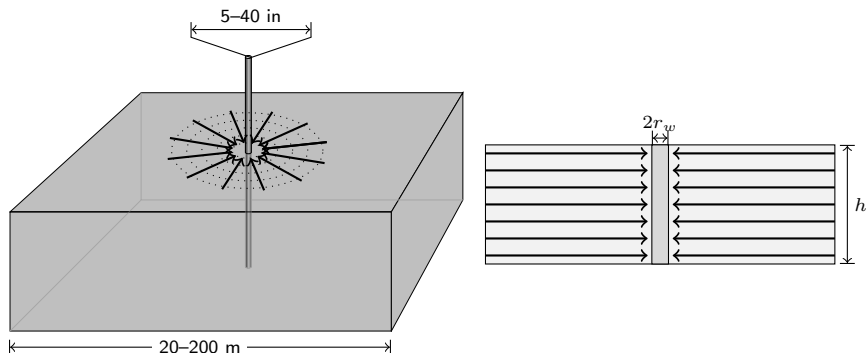
Inflow and outflow take place on a *subgrid* scale, with large variations in pressure over short distances.

Solution: use a linear inflow-performance relation

$$q = J\big(p_R - p_{bh}\big)$$

Here, p_{bh} is flowing pressure in wellbore and p_R average pressure in cell
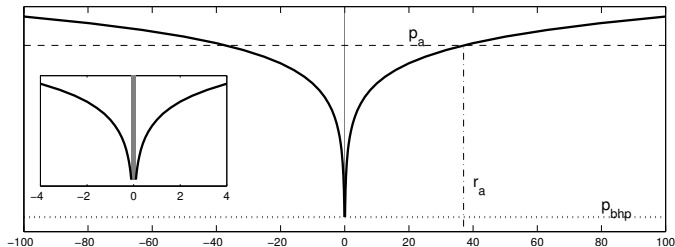
Wells: analytic subscale model

Pseudo-steady, radial flow. Mass conservation in cylinder coordinates
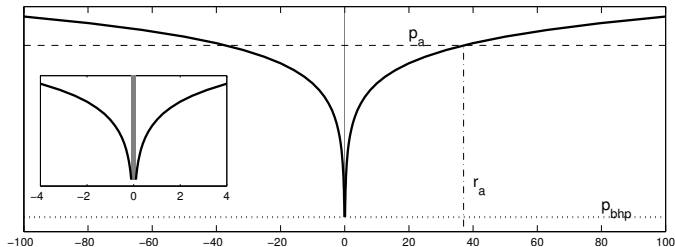
$$\frac{1}{r}\frac{\partial(ru)}{\partial r} = 0 \qquad \longrightarrow \qquad u = C/r.$$

Pseudo-steady, radial flow. Mass conservation in cylinder coordinates

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} = 0 \qquad \longrightarrow \qquad u = C/r.$$

Integrating around a small cylinder surrounding the well,

$$q = \oint \vec{u} \cdot \vec{n}\, ds = -2\pi h C$$

Wells: analytic subscale model
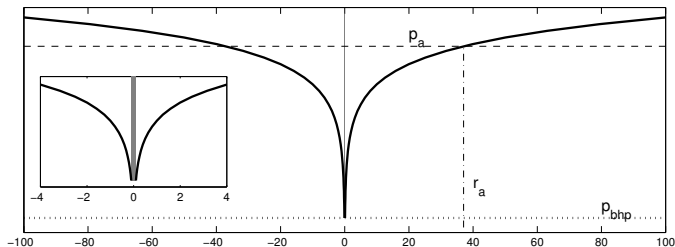
Insert into Darcy's law and integrate from wellbore radius r_w to drainage radius r_d at which $p = p_d$ is constant:

$$u = -\frac{q}{2\pi r h} = -\frac{K}{\mu}\frac{dp}{dr} \quad \longrightarrow \quad 2\pi K h \int_{p_{bh}}^{p_d} \frac{dp}{q\mu} = \int_{r_w}^{r_d} \frac{dr}{r}$$

Wells: analytic subscale model
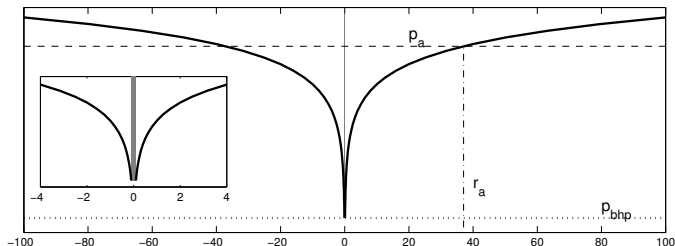
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Solution

$$q = \frac{2\pi K h}{\mu \ln(r_d/r_w)}\left(p_d - p_{bh}\right)$$

Wells: analytic subscale model
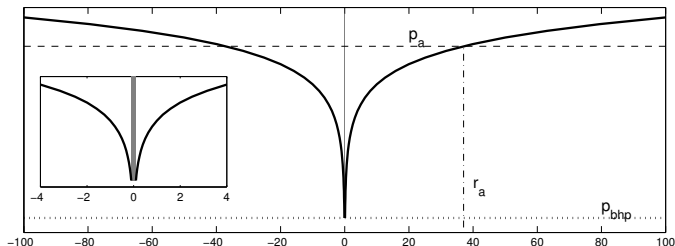
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Solution (volumetric average pressure $p = p_a$ at $r_a = 0.472 r_d$)

$$q = \frac{2\pi Kh}{\mu \ln(r_d/r_w)}(p_d - p_{bh}) = \frac{2\pi Kh}{\mu\big(\ln(r_d/r_w) - 0.75\big)}(p_a - p_{bh})$$

Wells: analytic subscale model
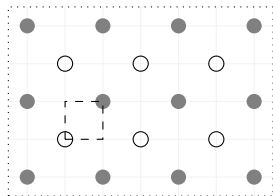
Producer
Injector
Quarter five-spot
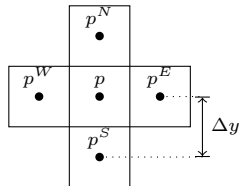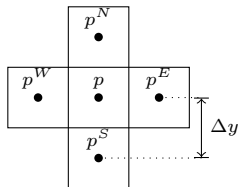
Repeated five-spot $\longrightarrow$ symmetric solution. Discretize Poisson's equation:

$$-\frac{Kh}{\mu}\left[4p - p^W - p^N - p^W - p^S\right] = q \qquad \longrightarrow \qquad p = p^E - \frac{q\mu}{4Kh}$$

Wells: analytic subscale model

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Analytic model valid in neighboring blocks

$$p = p_{bh} + \frac{q\mu}{2\pi Kh}\ln\left(\Delta x/r_w\right) - \frac{q\mu}{4Kh}$$

Wells: analytic subscale model

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Analytic model valid in neighboring blocks

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Wells: analytic subscale model
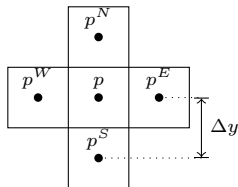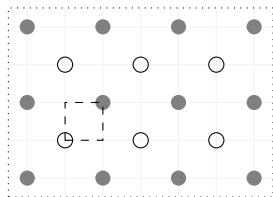

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Peaceman's formula:

$$q = \frac{2\pi Kh}{\mu \ln\big(r_e/r_w\big)}\big(p - p_{bh}\big), \qquad r_e = e^{-\frac{\pi}{2}}\sqrt{\Delta x \Delta y} \approx 0.20788\sqrt{\Delta x \Delta y}$$

Wells: many complications

There are several known extensions to Peaceman's well model:

- Diagonal permeability tensor $K \to \sqrt{K_x K_y}$
- Rectangular grid cells (more complex formula for r_e)
- Horizontal wells, off-centered wells, multiple wells, . . .
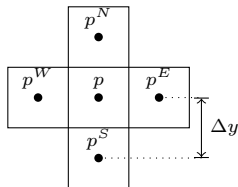- Near-well effects (permeability increase/reduction)
- Other grid types and discretization schemes

Despite obvious limiting assumptions, Peaceman's model is used rather uncritically in industry. Need for more accurate/robust/versatile models. . .

In general: need to describe flow within wellbore and annulus, downhole equipment, surface facilities, control strategies (choking, reinjection) involving complex logic, . . .

Multisegment wells

More accurate modelling:

– Network models

– Represent annulus

– Flow inside wellbore

– (Autonomous) inflow control devices

– Artificial lift, etc
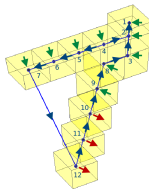
In nodes:

– pressure p

– mass fractions x_w^m, x_o^m, x_g^m

In segments:

– mass rates v^m

Multisegment wells

More accurate modelling:

– Network models

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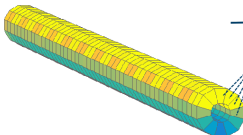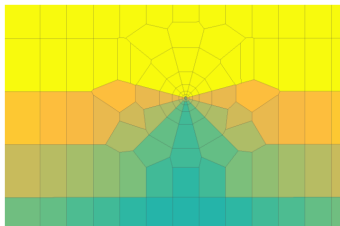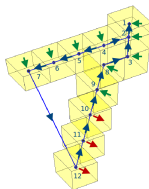– mass rates v^m

Discrete mass conservation in nodes:

$$\underbrace{\frac{V}{\Delta t}\big(x_c\rho - x_c^0\rho\big)}_{\text{accumulation}} \quad + \quad \underbrace{\operatorname{div}\big(v_c^m\big)}_{\substack{\text{in- and outflux to}\\\text{neighboring nodes}}} \quad - \quad \underbrace{q_c^m}_{\substack{\text{source term (well}\\\text{control, connections)}}} \quad = 0$$

Discrete pressure drop equations in segments:

$$\operatorname{grad}(p) \quad - \quad \underbrace{g\operatorname{avg}(\rho)\operatorname{grad}(z)}_{\text{gravity term}} \quad - \quad \underbrace{h\big(v^m, \operatorname{uw}(\rho), \operatorname{uw}(\mu)\big)}_{\substack{\text{heuristic pressure}\\\text{drop term}}} = 0$$

Outline

Black-oil: discretization and linearization

Discretization: backward Euler in time, two-point flux-approximation with upstream mobility in space.

Newton's method for nonlinear equation: $\dfrac{\partial E}{\partial x^i}\delta x^i = E(x^i)$
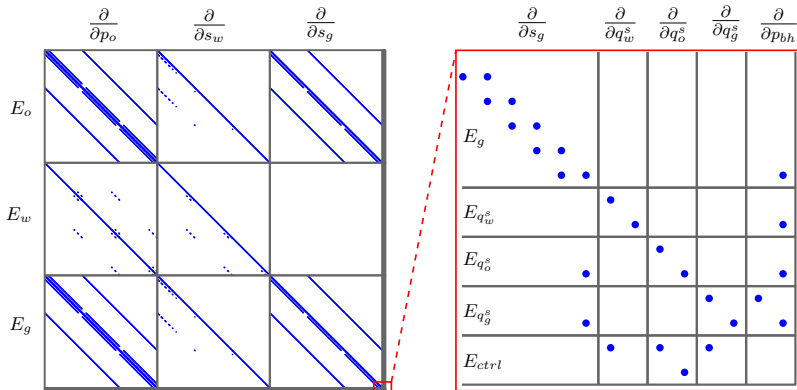
Black-oil: solution strategies

Solution procedure

1. Eliminate well variables q_o^s, q_w^s, q_g^s, and p_{bh}

2. Set first block-row equal to sum of block-rows, leave out rows that may harm diagonal dominance in block (1,1)

3. Set up two-stage preconditioner:
   - $\mathbf{M}_1^{-1}$: solves pressure subsystem
   - $\mathbf{M}_2^{-1}$: ILU0 decomposition of the full system

4. Solve full system with GMRES using preconditioner $\mathbf{M}_2^{-1}\mathbf{M}_1^{-1}$

5. Recover remaining variables

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5. Recover remaining variables

For larger models, pressure subsystem should be solved with algebraic multigrid

Time-step control

- chop if too large changes in variables
- chop if convergence failure
- more advanced logic to maintain targeted iteration count

Elaborate logic for well control and surface facilities

Example: SPE 9 benchmark

- Grid with 9000 cells
- 1 water injector, rate controlled, switches to bhp
- 25 producers, oil-rate controlled, most switch to bhp
- Appearance of free gas due to pressure drop
- Production rates lowered to 1/15 between days 300 and 360
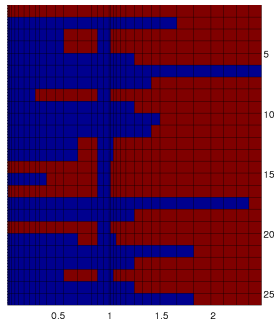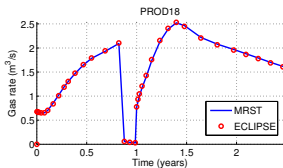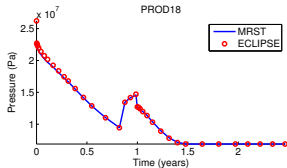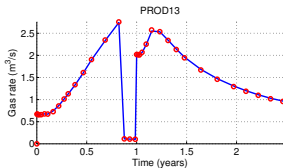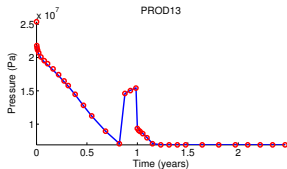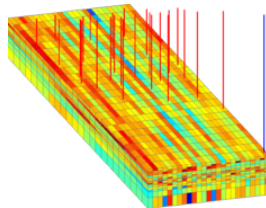
Example: the Voador field
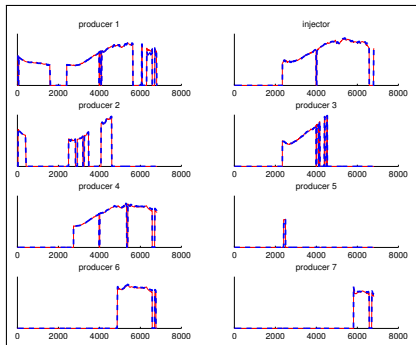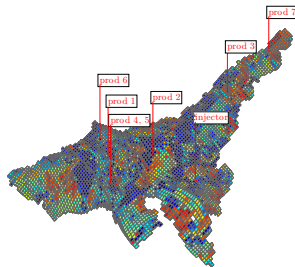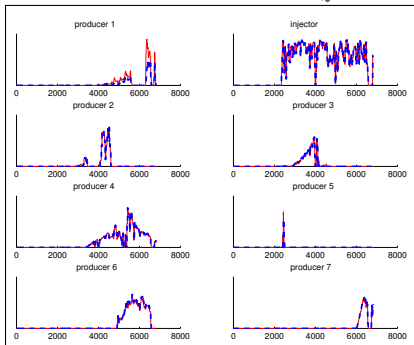
- South wing of the reservoir (Petrobras)
- Gradients obtained through adjoint simulations
- Validate: open-source / commercial simulator:
  - 20 years of historic data
  - virtually identical results
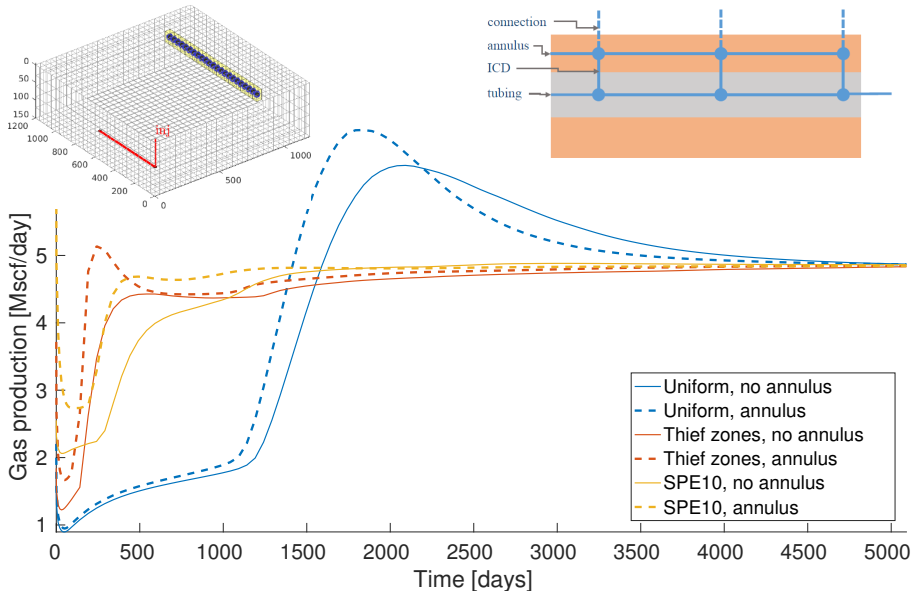 - main challenge: needed to reverse-engineer description of wells…
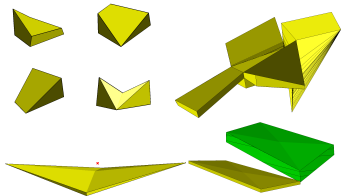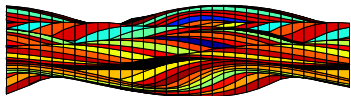

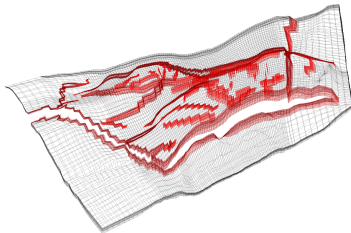bottom-hole pressure

water rate

Example: effect of modeling annulus

Summary

- Geological models: complex unstructured grids having many obscure challenges
- Flow models: system of highly nonlinear parabolic PDEs with elliptic and hyperbolic sub-character
- Well models: subscale models, complex logic, strong impact on flow
- Validation and availability in software

Challenges:

- Main point of grid: describe stratigraphy and structural architecture, i.e., not chosen freely to maximize accuracy of numerical discretization
- Industry standard: corner-point / stratigraphic grids
- Grid topology is generally unstructured, with non-neighboring connections
- Geometry: deviates (strongly) from box shape, high aspect ratios, many faces/neighbors, small faces, . . .
- Potential inconsistencies since faces are bilinear or tetrahedral surfaces
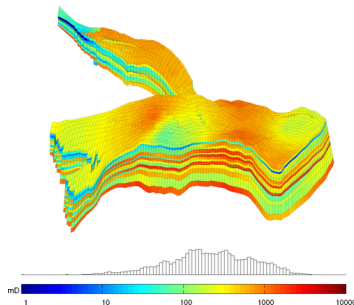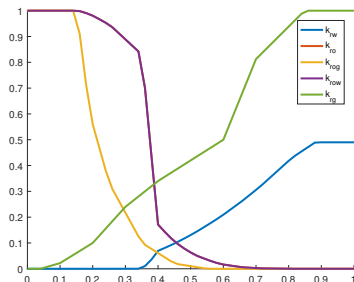
Summary

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Challenges:

- Delicate balances: viscous forces, gravity, capillary, . . .
- Strong coupling between 'elliptic' and 'hyperbolic' variables (small scale: capillary, large scale: gravity)
- Large variation in time constants and coupling
- Orders-of-magnitude variations in permeability
- Parameters with discontinuous derivatives
- Path-dependence: hysteretic parameters
- Sensitive to subtle changes in interpolation of tabulated physical data
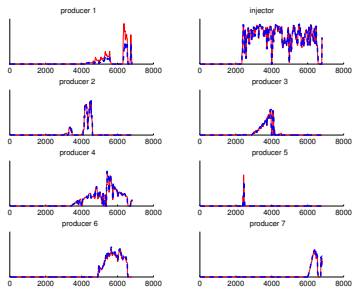- Monotonicity and mass conservation

Summary

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- Validation and availability in software

Challenges:

- Near singular radial flow in near-well zone (much larger flow than inside reservoir)

- Induce nonlocal connections

- Completely different multiphase flow inside wellbore

- Coupling to surface facilities

- Abrupt changes in driving forces

- Control strategies with intricate logic which is highly sensitive to state values
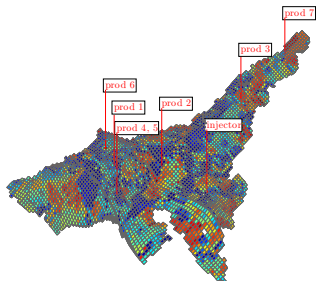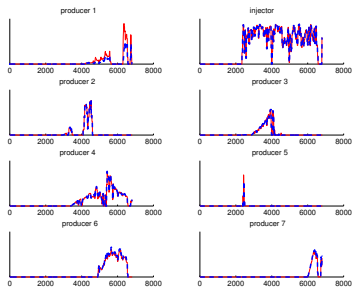
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- Validation and availability in software

Challenges:

- New methods tend to be immature and too simplified
- Researchers: incompressible flow and explicit methods. Industry: implicit methods for compressible flow
- Industry relies on a few software providers and has strong faith in software with (undocumented) safeguards and algorithmic choices
- Oil companies seldom give away data
- Realistic models involve a large number of intricate details (Eclipse has 2–3000 keywords. . .)